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Geometry and Calibration

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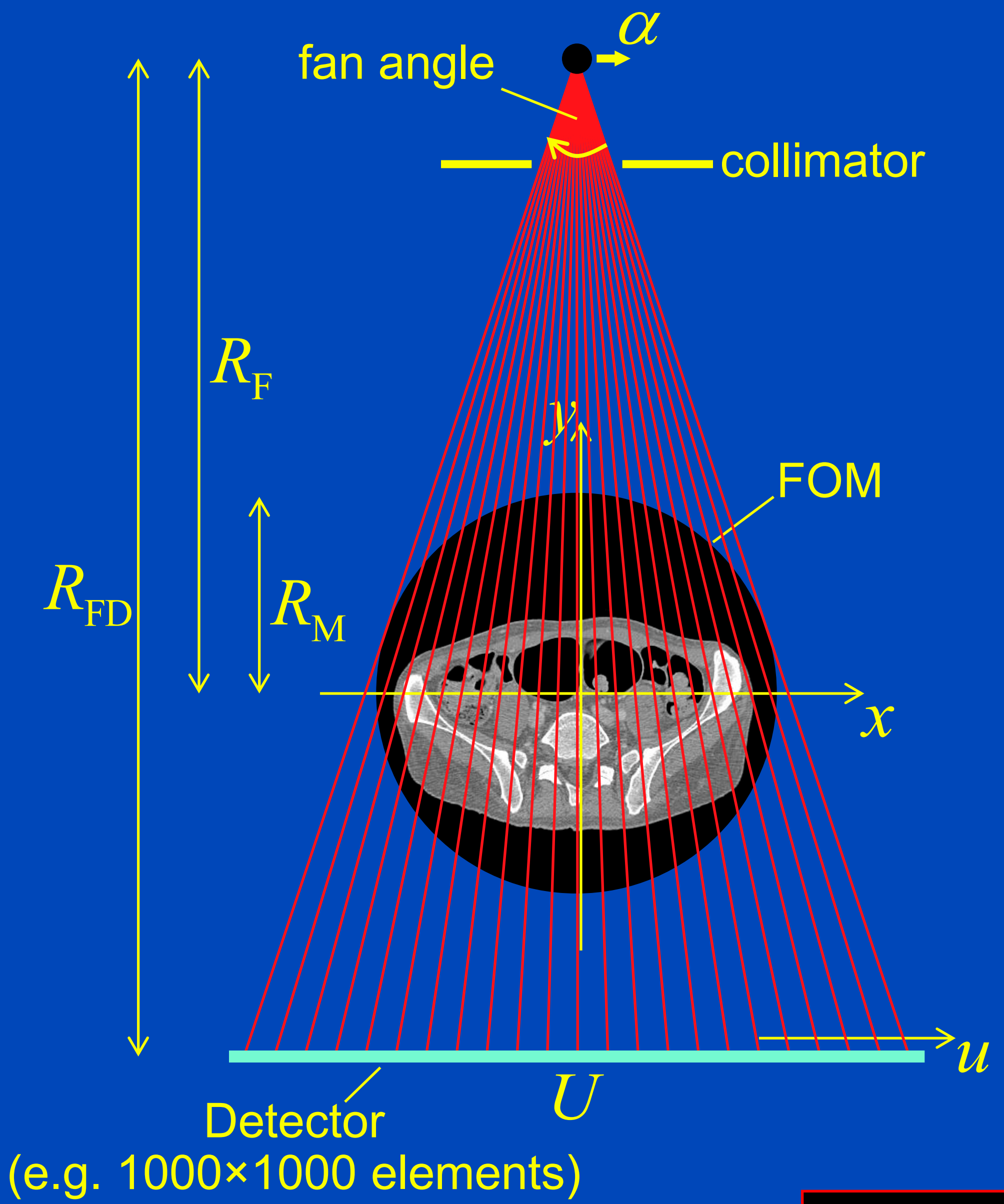
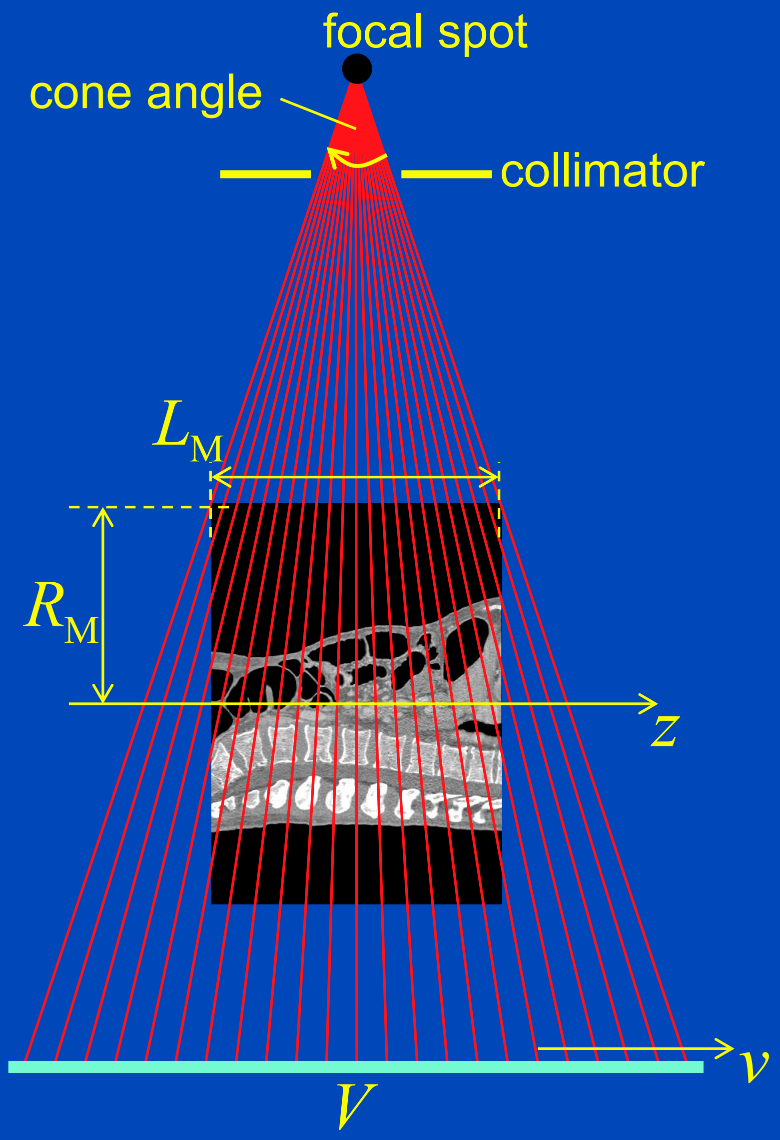
DEUTSCHES
KREBSFORSCHUNGSZENTRUM
IN DER HELMHOLTZ-GEMEINSCHAFT



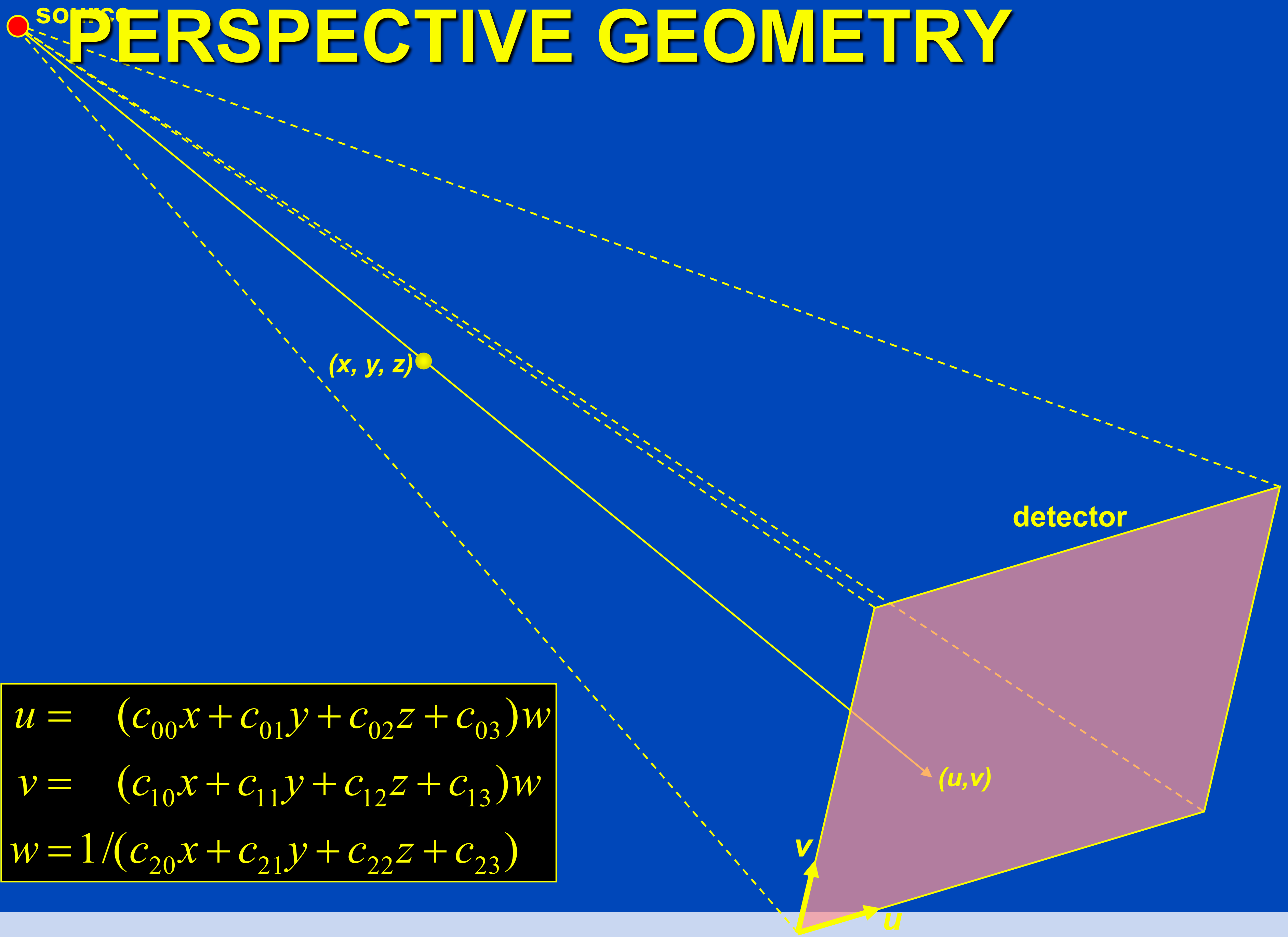


$$2R_M / U > L_M / V$$

$$R_M = R_F / \sqrt{1 + (2R_{FD} / U)^2}$$

$$L_M = V(R_F - R_M) / R_{FD}$$

PERSPECTIVE GEOMETRY

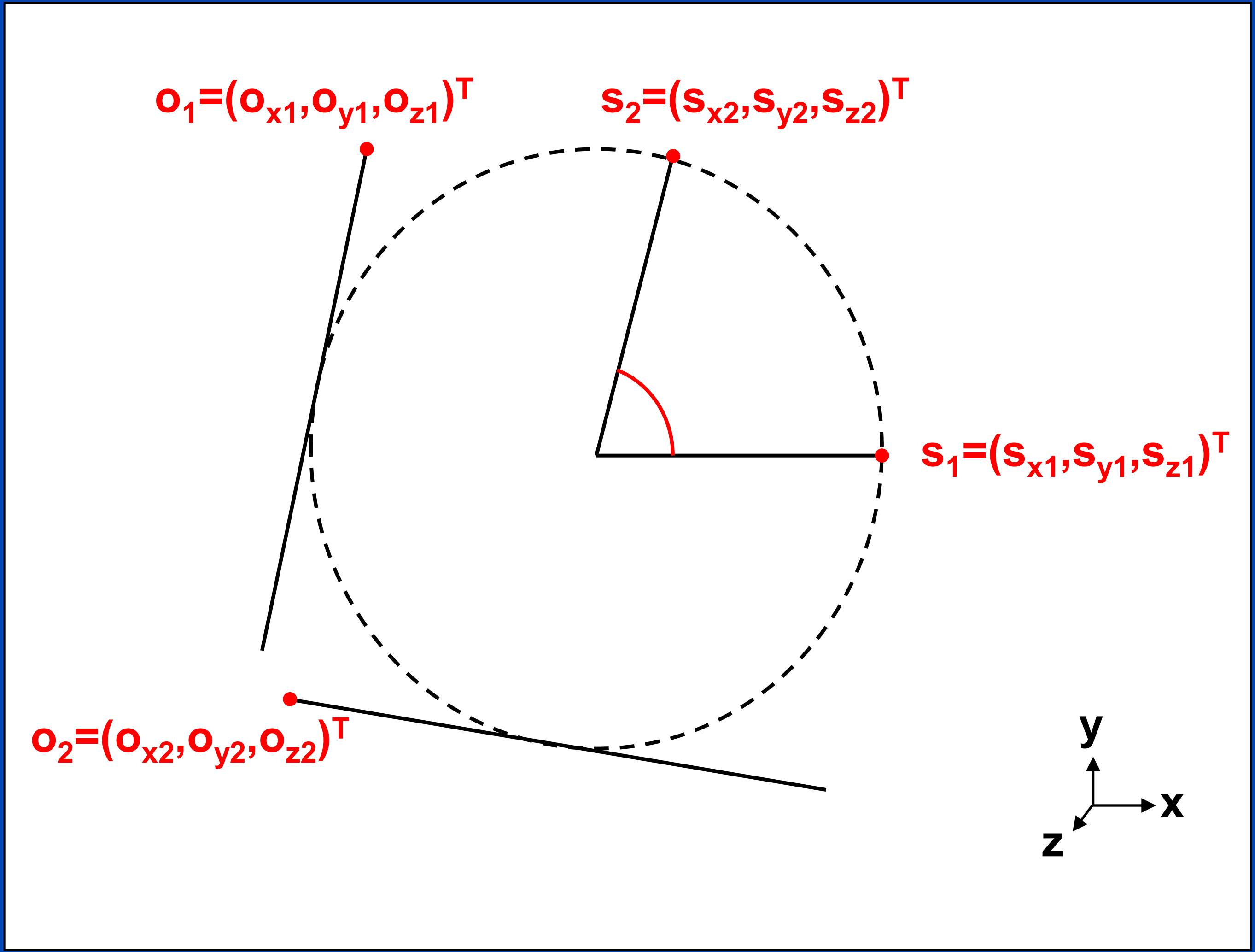


$$u = (c_{00}x + c_{01}y + c_{02}z + c_{03})w$$
$$v = (c_{10}x + c_{11}y + c_{12}z + c_{13})w$$
$$w = 1/(c_{20}x + c_{21}y + c_{22}z + c_{23})$$

Bead positions unknown, trajectory perfectly known

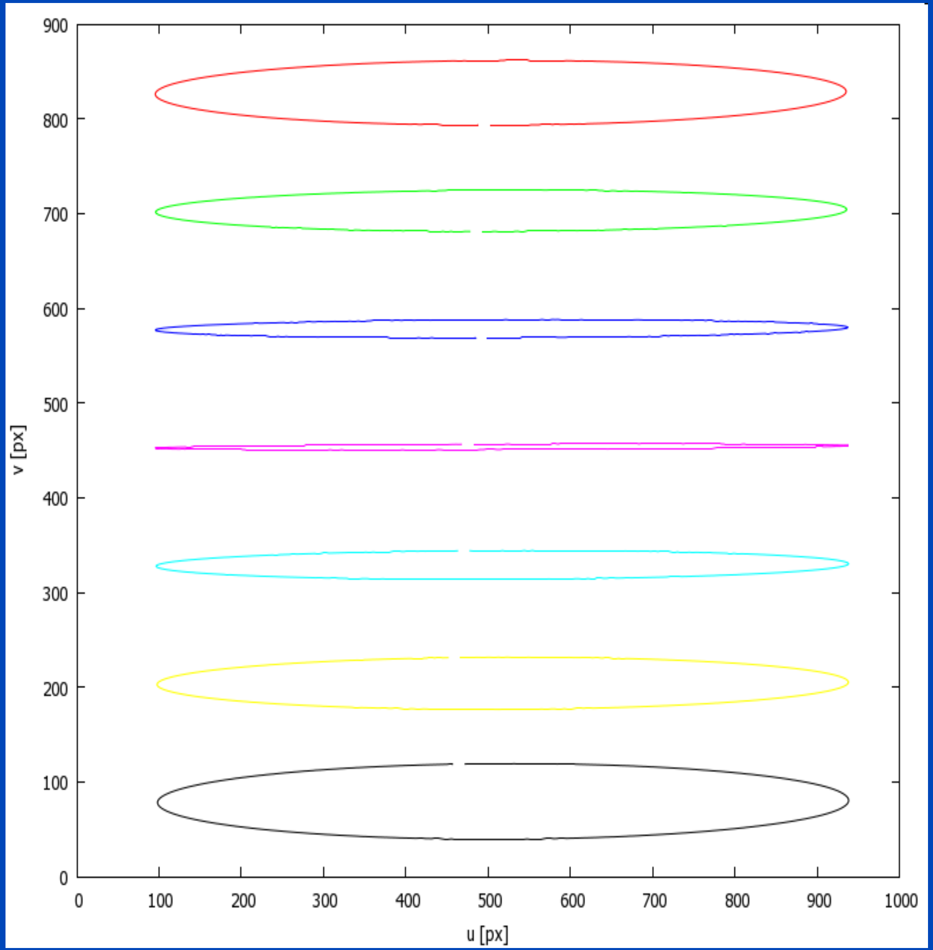
GEOMETRY FROM BEADS

Geometry Definition (Dual Source Micro-CT)



Calibration Procedure

- Acquire a sequence scan of several metal beads/spheres
- Segment the center of mass in each projection (or do better, see later slides)
- Find a geometry and sphere positions that fit the measurements



A robust geometry estimation method for spiral, sequential and circular cone-beam micro-CT

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Purpose: The authors propose a novel method for misalignment estimation of micro-CT scanners using an adaptive genetic algorithm.
Methods: The proposed algorithm is able to estimate the rotational geometry, the direction vector of table movement and the displacement between different imaging threads of a dual source or even multisource scanner. The calibration procedure does not rely on dedicated calibration phantoms and a sequence scan of a single metal bead is sufficient to geometrically calibrate the whole imaging system for spiral, sequential, and circular scan protocols. Dual source spiral and sequential scan protocols in micro-computed tomography result in projection data that—besides the source and detector positions and orientations—also require a precise knowledge of the table direction vector to be reconstructed properly. If those geometric parameters are not known accurately severe artifacts and a loss in spatial resolution appear in the reconstructed images as long as no geometry calibration is performed. The table direction vector is further required to ensure that consecutive volumes of a sequence scan can be stitched together and to allow the reconstruction of spiral data at all.
Results: The algorithm's performance is evaluated using simulations of a micro-CT system with known geometry and misalignment. To assess the quality of the algorithm in a real world scenario the calibration of a micro-CT scanner is performed and several reconstructions with and without geometry estimation are presented.
Conclusions: The results indicate that the algorithm successfully estimates all geometry parameters, misalignment artifacts in the reconstructed volumes vanish, and the spatial resolution is increased as can be shown by the evaluation of modulation transfer function measurements. © *2012 American Association of Physicists in Medicine*. [http://dx.doi.org/10.1118/1.4739506]

Calibration Procedure

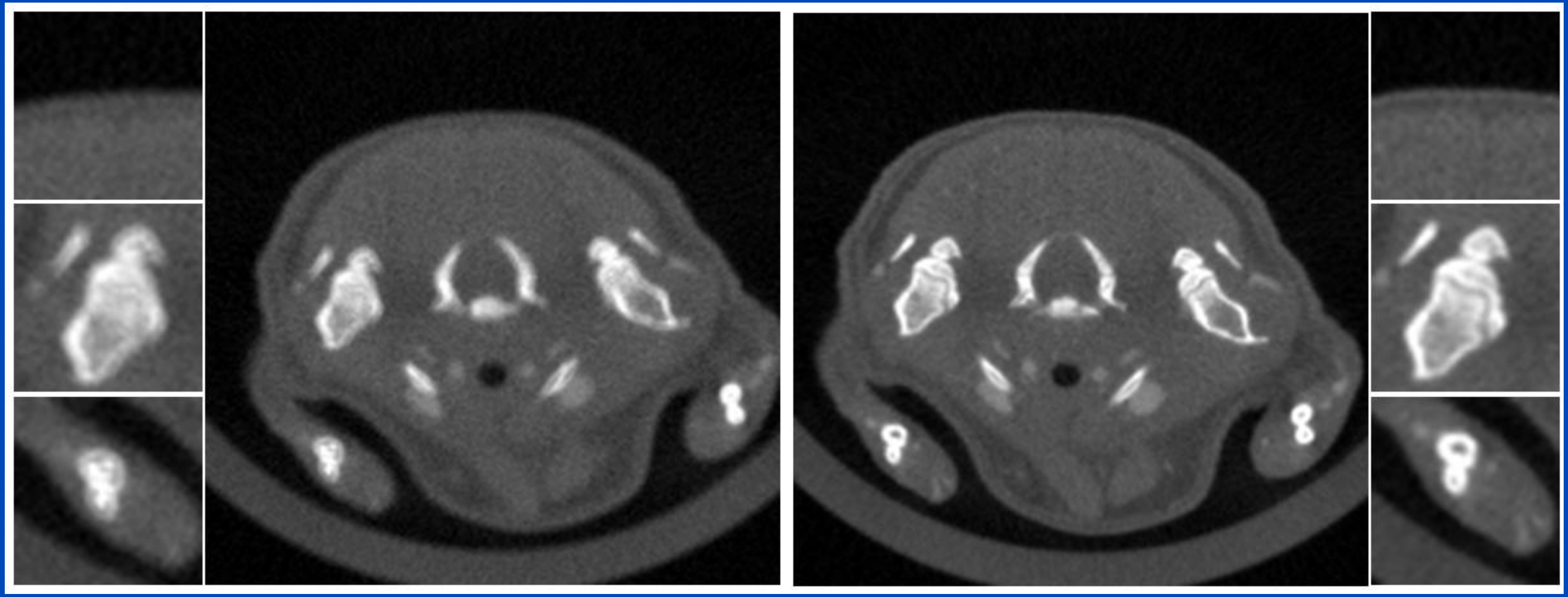
- Finding a geometry and bead positions that fit the measurements is equivalent to solve:

$$E^2 = \sum_i \left(\left(u_i(\Omega) - \hat{u}_i \right)^2 + \left(v_i(\Omega) - \hat{v}_i \right)^2 \right)$$
$$\Omega = (s_x, s_y, s_z, o_x, o_y, o_z, \ldots, r_0, \ldots r_{N-1})$$

- This cost function obeys several local optima thus a global optimum has to be found
- Literature shows that optimization can not be done by common numerical methods, e.g. Levenberg-Marquardt so we use an adaptive genetic algorithm

Results

No Calibration | With Calibration



Estimation of spheres and sphere segments in projections

ESTIMATION OF BEAD CENTERS

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Phys. Med. Biol. **62** (2017) 6486–6496 — https://doi.org/10.1088/1361-6560/aa7a96

Model-based sphere localization (MBSL) in x-ray projections

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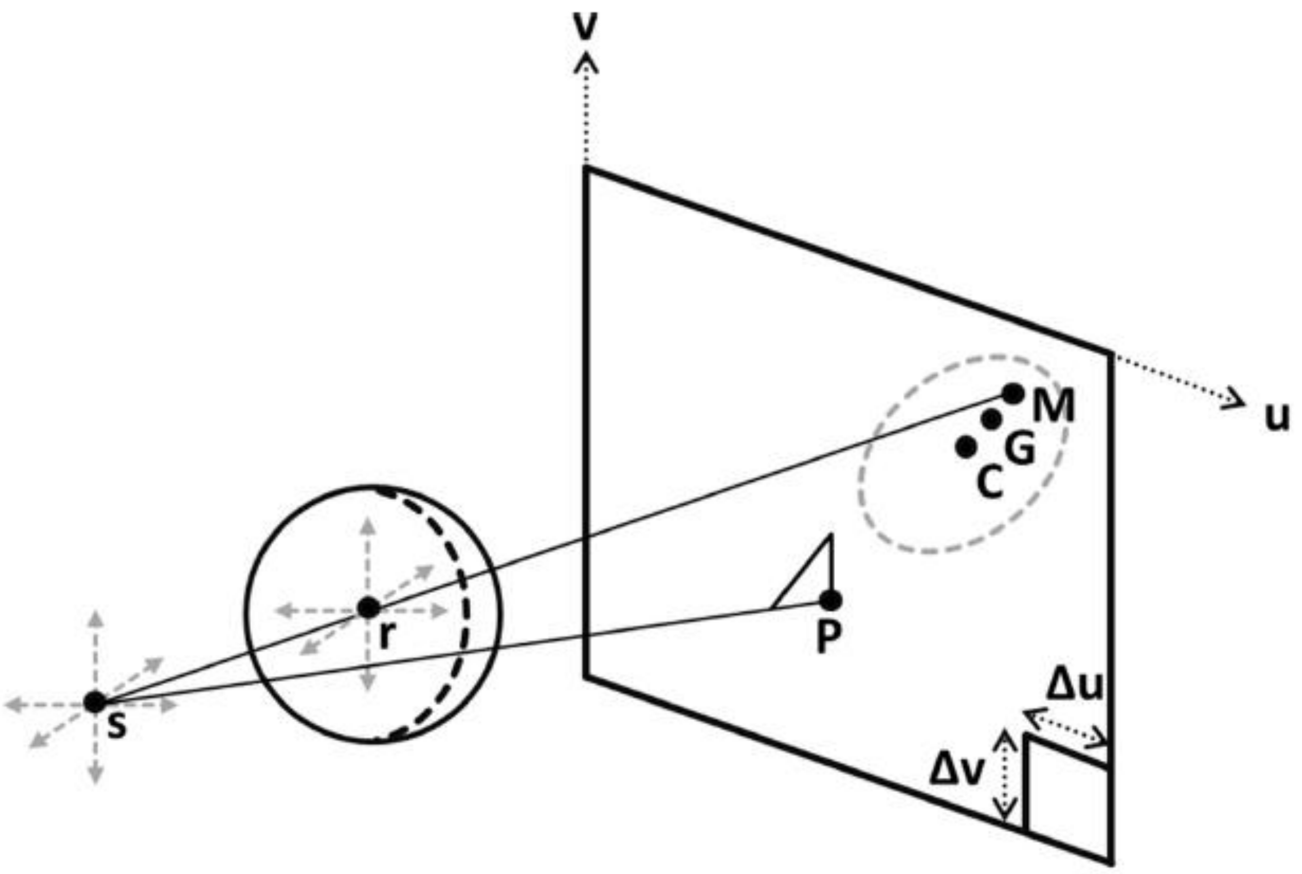
CrossMark

Abstract

The detection of spherical markers in x-ray projections is an important task in a variety of applications, e.g. geometric calibration and detector distortion correction. Therein, the projection of the sphere center on the detector is of particular interest as the used spherical beads are no ideal point-like objects. Only few methods have been proposed to estimate this respective position on the detector with sufficient accuracy and surrogate positions, e.g. the center of gravity, are used, impairing the results of subsequent algorithms. We propose to estimate the projection of the sphere center on the detector using a simulation-



Aim

- We seek to minimize the weighted difference between the measured projections $q(u, v)$ and simulated sphere projections $p(r, s, u, v)$ to find the projection of the sphere center r.

$$\min_{\boldsymbol{c},\boldsymbol{r},\boldsymbol{s}} \int du\, dv\, \frac{1}{\sigma^2(u,v)} \Big(\sum_n c_n q^n(u,v) - p(\boldsymbol{r},\boldsymbol{s},u,v) \Big)^2$$

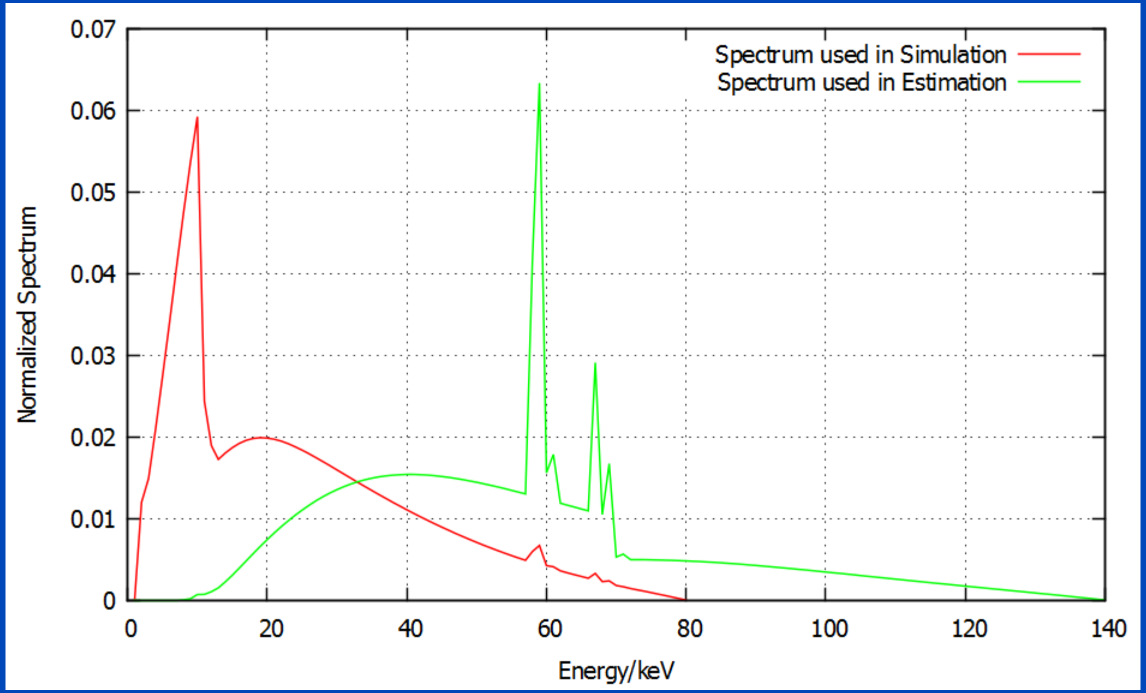
- This also includes finding the focal spot position s and the sphere center r (nearby their assumed positions), as well as the beam hardening polynomial coefficients c.
- The sphere radius is well-known, but would automatically be absorbed when determining c, r and s.

Simulation Parameters

- We use a simple flat detector geometry:
  - R_F = 570 mm
  - R_D = 380 mm
  - $du = dv$ = 0.4 mm
  - $Nu = Nv$ = 1024
- A sphere with diameter 6 mm made of steel (Fe70Cr30) is simulated at 46×46 different positions on the detector. Hence, the deviation maps show the deviation between estimated and simulated position for these 2116 positions.
- Furthermore, the simulated detector values were disturbed using Gaussian noise with a FWHM of 0.5 mm.

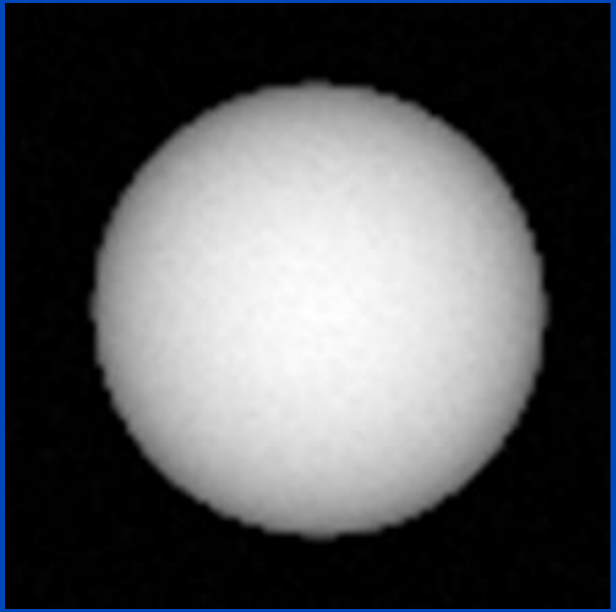
Simulation Parameters

- The sphere projections were generated using the Tucker tungsten spectrum. To simulate an unknown spectrum, we distinguish between the spectrum used for simulation and the spectrum used in the estimation procedure:
- Simulation:
  - 80 kV
  - Anode angle of 12°
  - Characteristic peaks and bremsstrahlung
- Estimation:
  - 140 kV
  - Anode angle of 6°
  - Characteristic peaks and bremsstrahlung
  - 0.5 mm Al prefiltration

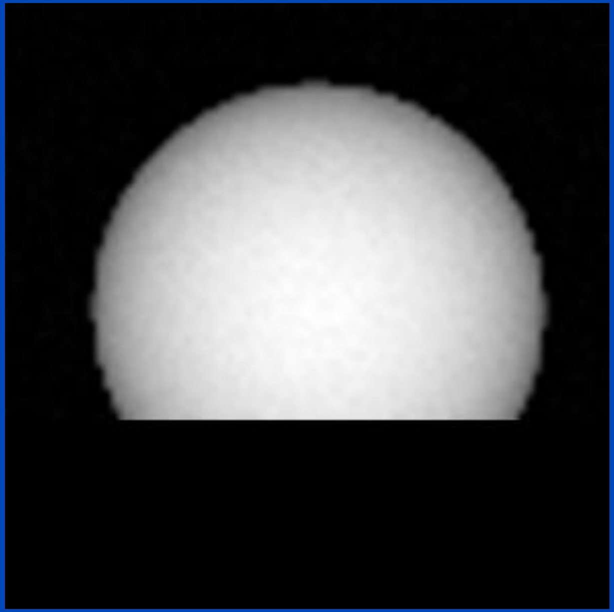


Clipping

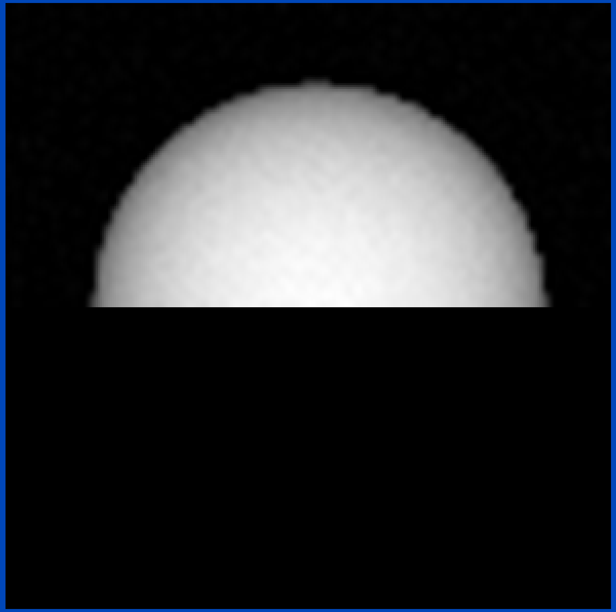
- As the metal beads are usually connected to a holder, only a segment of the sphere can be used for estimation.
- This is modelled by clipping the sphere in v-direction between the real center and the maximum extent in v-direction.



0% clipping



25% clipping

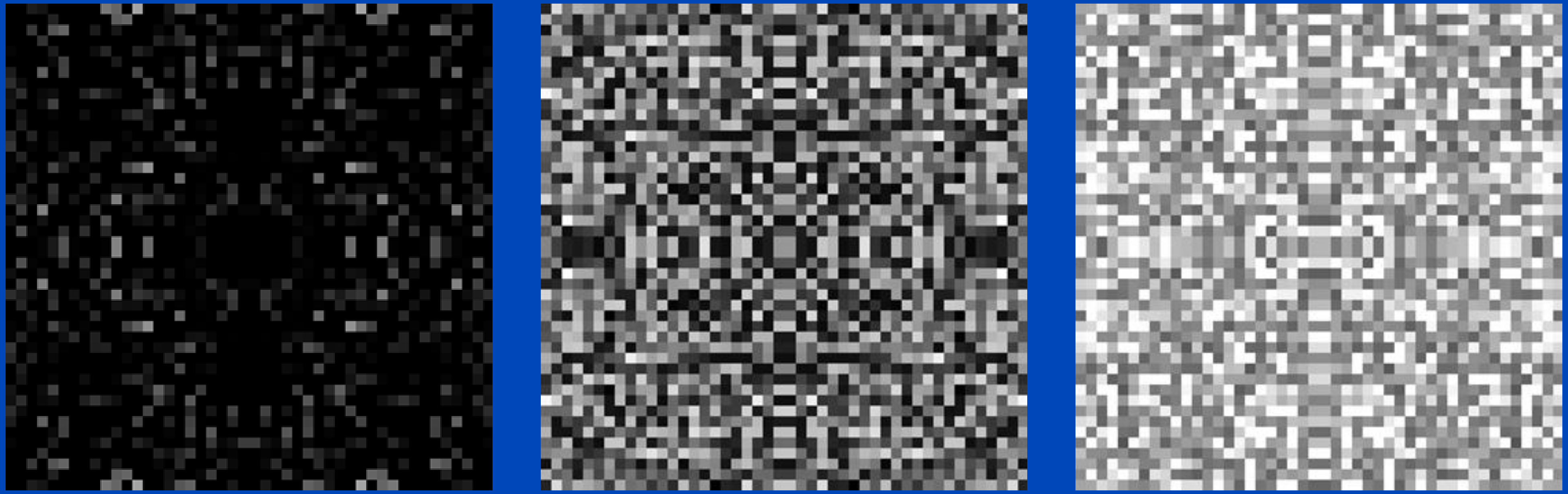


50% clipping

Absolute Position Error

With Noise, Proposed Method (q[1])

0% Clipping | 25% Clipping | 50% Clipping

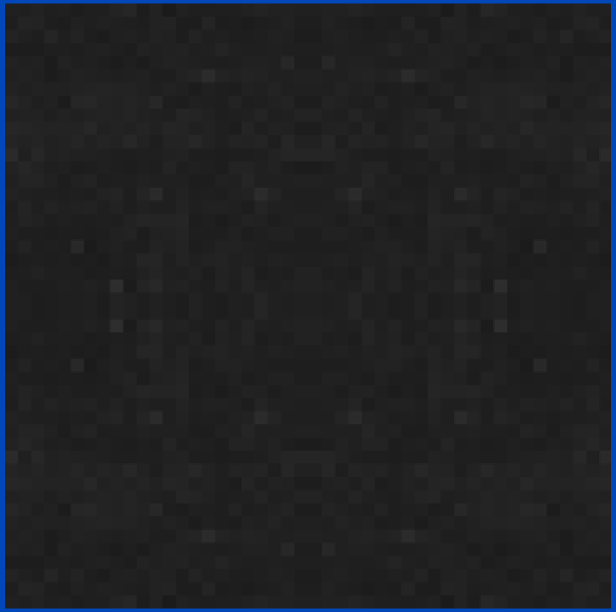


C = 1500 µm, W = 3000 µm

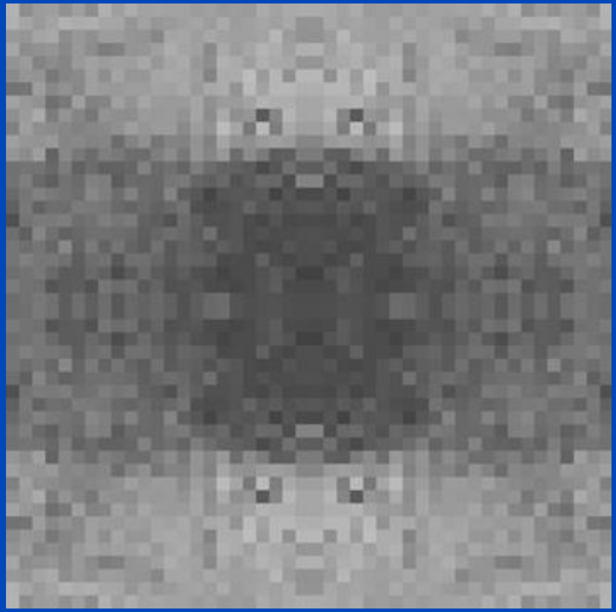
Results

With Noise, Proposed Method ($c_0q^0+c_1q^1+c_2q^2$)

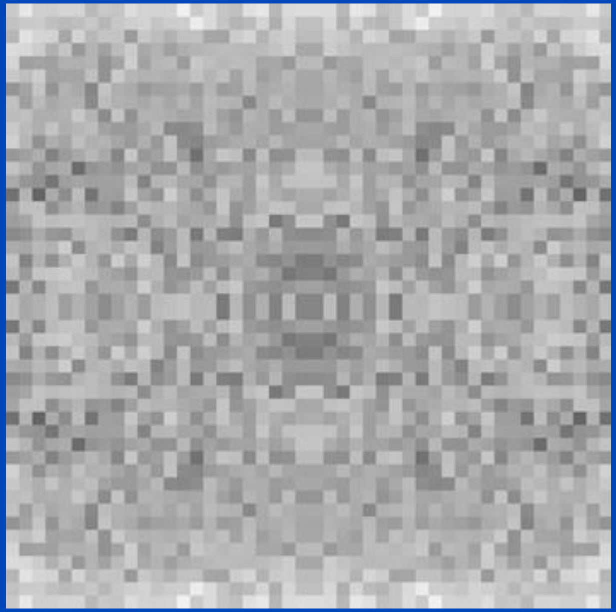
0% Clipping



25% Clipping



50% Clipping



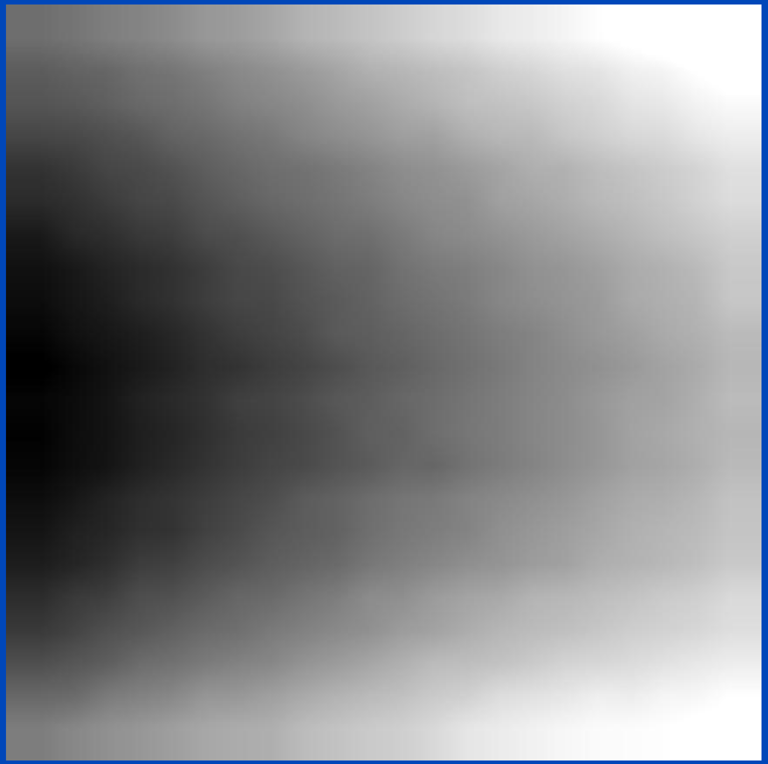
C = 75 µm, W = 150 µm

HOW FLAT ARE FLAT DETECTORS?

Example

Deviation in *u*

Deviation in *v*



C = -0.3 px, W = 1 px

Remove Perspective Component

- Distortions may include misaligned perspective geometry.
- Use perspective model

$$\hat{T}_u(u,v) = \frac{c_{00}u + c_{01}v + c_{02}}{c_{20}u + c_{21}v + 1}$$

$$\hat{T}_v(u,v) = \frac{c_{10}u + c_{11}v + c_{12}}{c_{20}u + c_{21}v + 1}$$

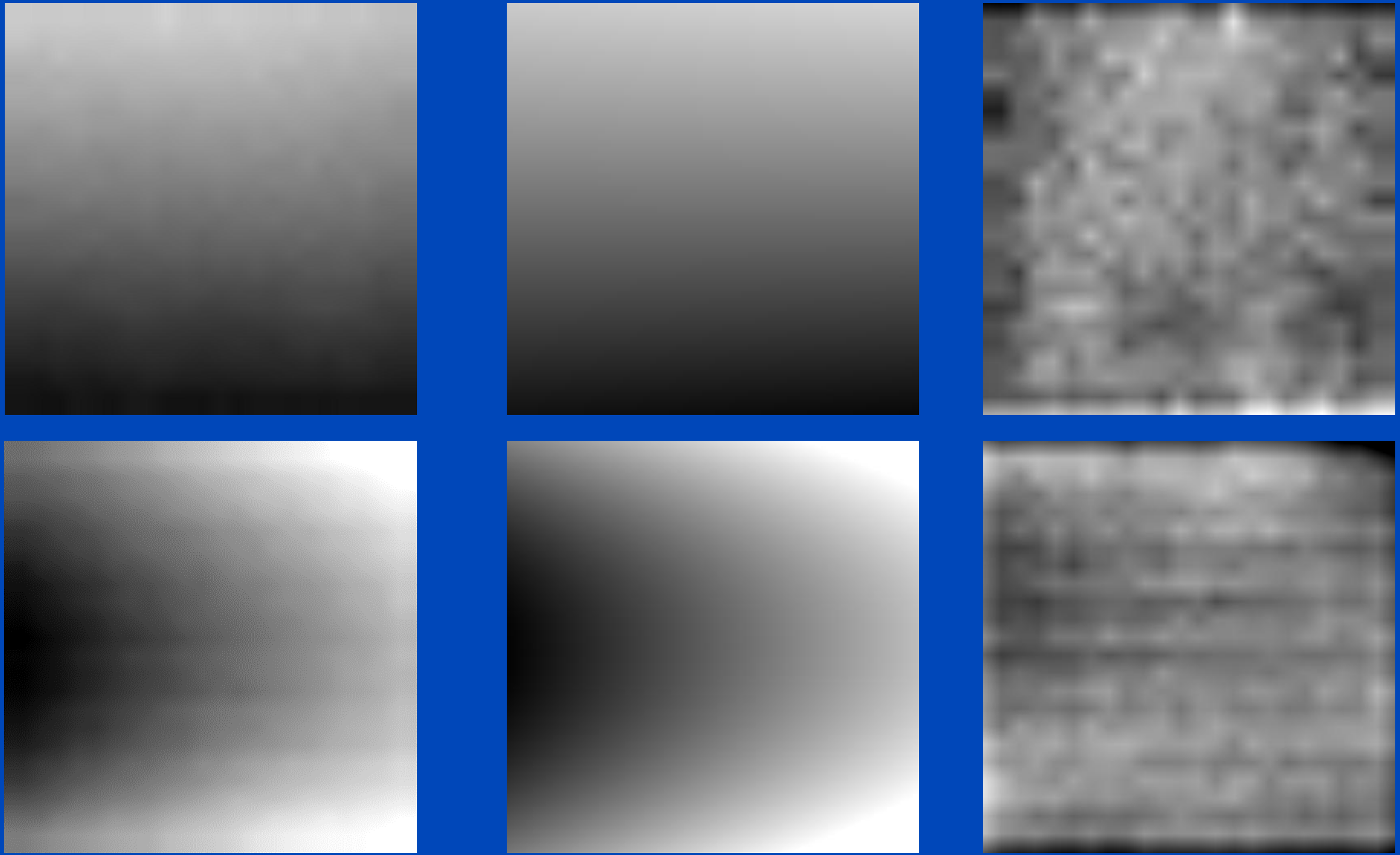
for *u* and for *v* distortion to find c_{ij} that minimize the distortions.

Results (Detector 1)

Measurement | **Perspective** | **Distortion**

Deviation in *u*

Deviation in *v*



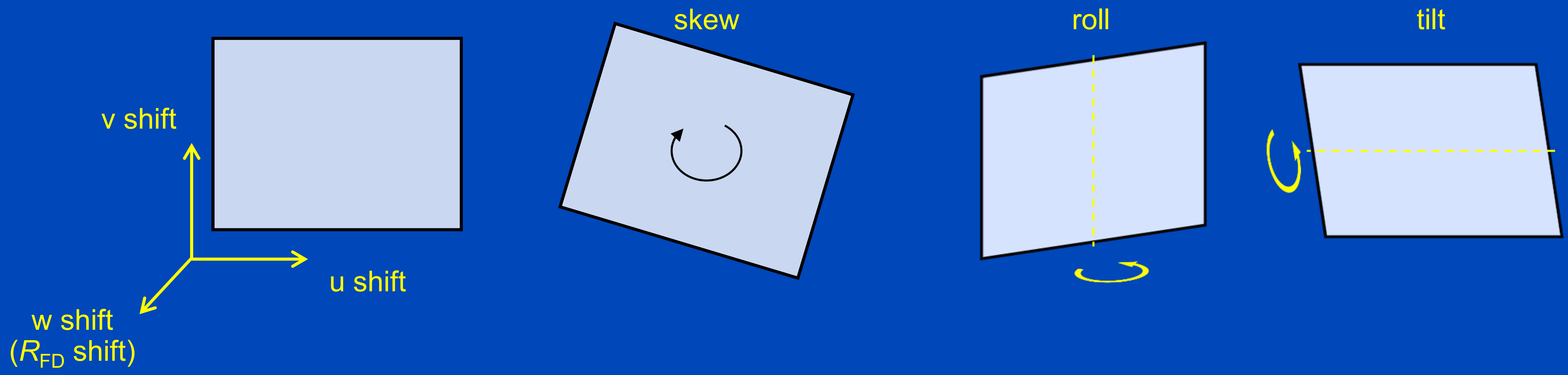
C = -0.3 px, W = 1 px | C = -0.3 px, W = 1 px | C = 0 px, W = 0.4 px

Intrinsic Calibration

MENG‘S METHOD

Aim

- Meng et al. proposed a simple method to estimate misalignment parameters from the sum of the acquired projections.
- In their publication, all misalignment parameters were estimated while accurate results were only derived for u-shift, skew and roll.
- In the following the accuracy of Meng's method to determine misalignment parameters is evaluated.
- Evaluation of the influence of beam hardening and truncation on the estimation.



Method

- For a given object $f(r, \alpha, z)$ the sum of projections SOP is given by:

$$\mathrm{SOP}(u, v) = \int X_{\alpha'}[f(r, \alpha, z)]d\alpha' \qquad (1)$$

- where X_α is the x-ray transform for the angle α:

$$X_\alpha f(r, \alpha, z) = p_\alpha(u, v)$$

- A rotation of source and detector is similar to a rotation of the object. Thus (1) can be written as:

$$\mathrm{SOP}(u, v) = \int X_0[f(r, \alpha - \alpha', z)]d\alpha'$$

- Using the linearity of the x-ray transform:

$$\mathrm{SOP}(u, v) = X_0 \int [f(r, \alpha - \alpha', z)]d\alpha'$$

Implementation

- Provide an ideal geometry ($\text{souvVec}_{\text{Ideal}}$) and a set of projections that was acquired over an angular range of 360°. (If the real measurement geometry differs from that ideal geometry, misalignment artifacts are introduced to the reconstruction.)
- The sum of the provided projections is calculated.
- For every geometry estimate ($\text{souvVec}_{\text{Real}}$) a rebinning of SOP from real to ideal is performed and the following cost function is evaluated

$$C = \int du\, dv \left(\mathrm{SOP}_{\mathrm{rebinned}}(u,v) - \mathrm{SOP}_{\mathrm{rebinned}}(-u,v)\right)^2$$

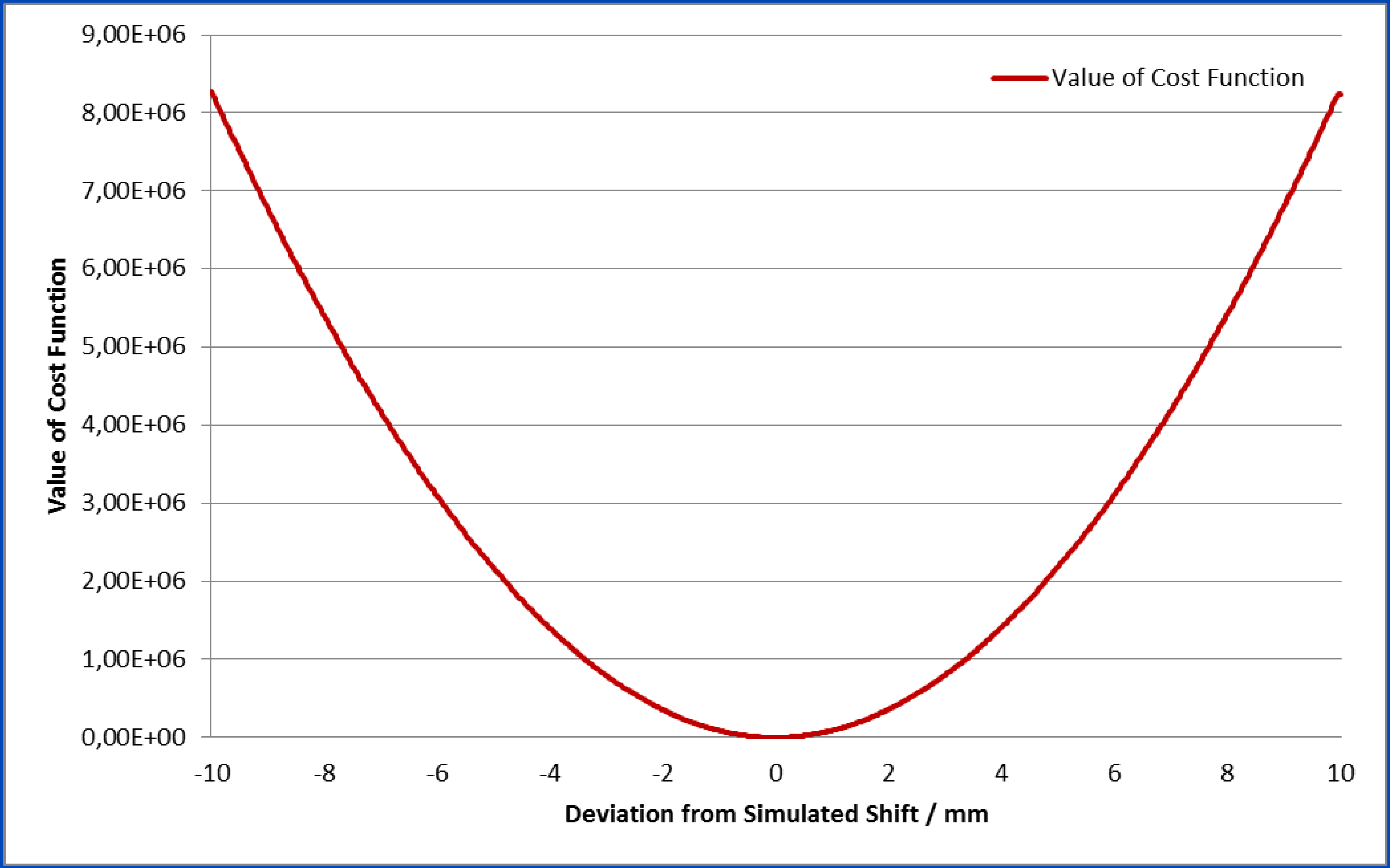
- The geometry that minimizes the cost function is assumed to be the actual measurement geometry.

Simulations (Misalignment)

- Definition of an ideal geometry
- Simulation of projections of a water sphere with a corrupted geometry. The sphere is placed off-center to avoid a rotational symmetry.
- For every simulation only one of the misalignment parameters (u-shift, v-shift, w-shift, skew, roll, tilt) differs from the ideal geometry
- Evaluation of the cost function as well as the reconstruction results

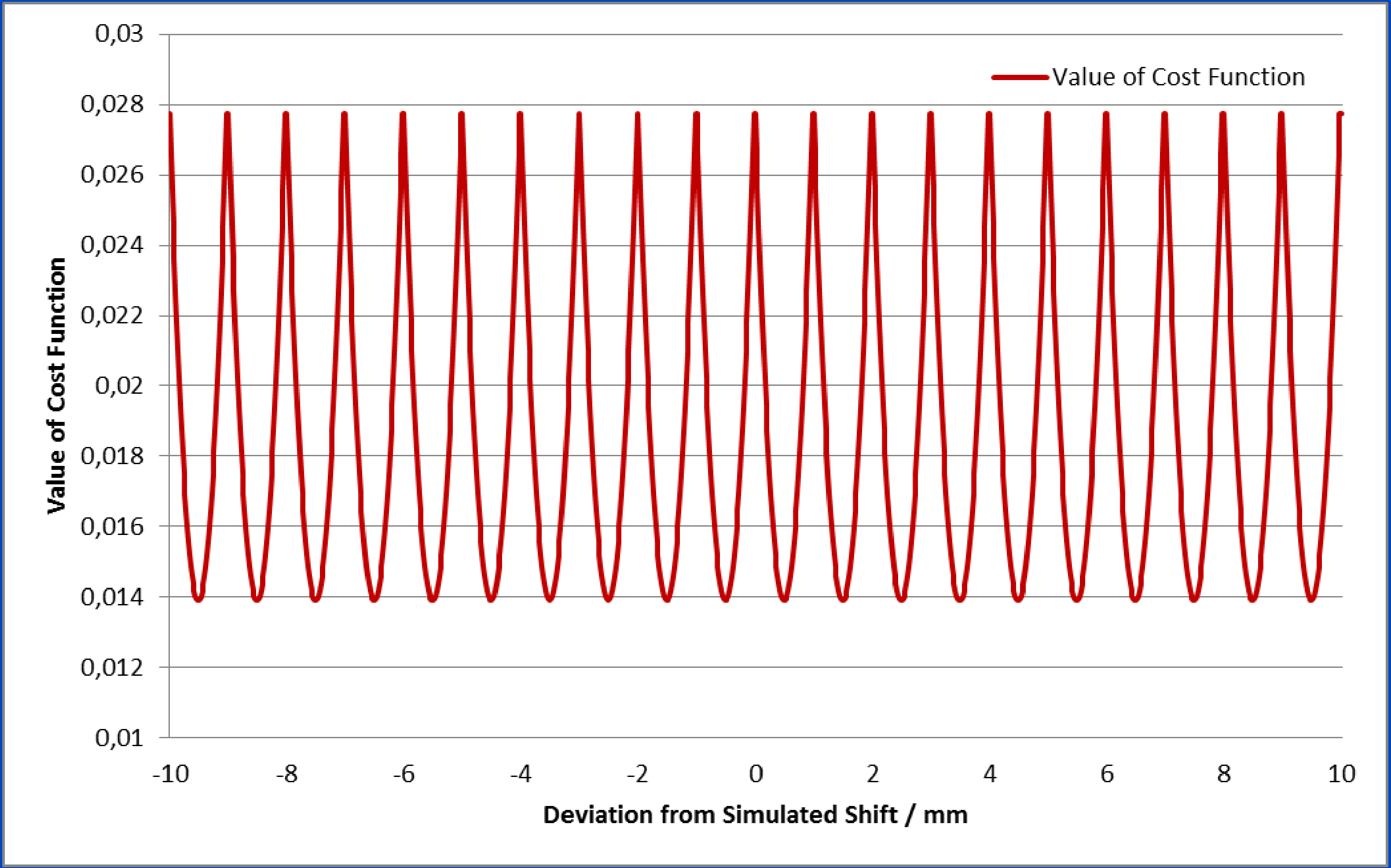
Results: Cost Function

u-Shift



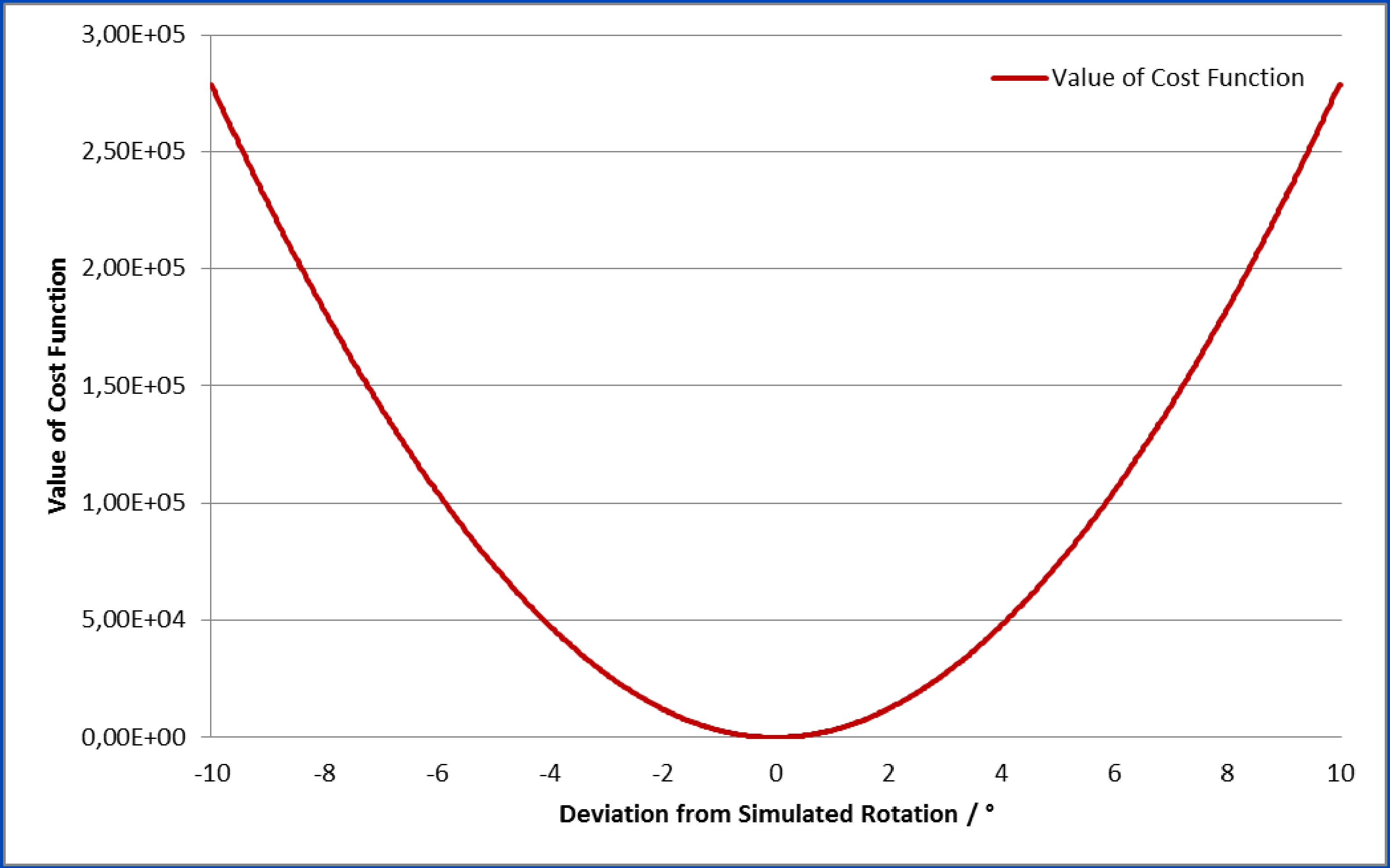
Results: Cost Function

v-Shift



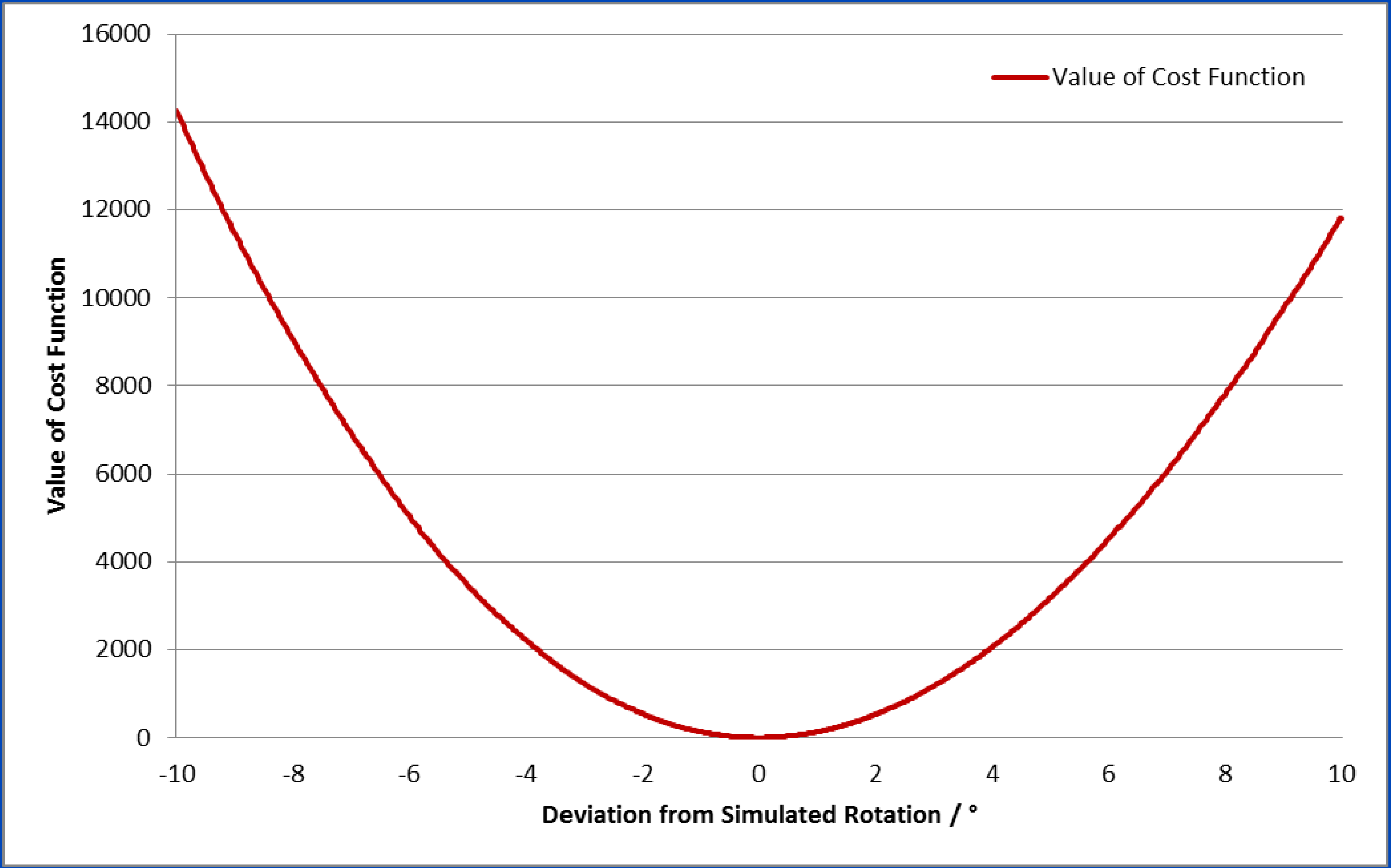
Results: Cost Function

Skew



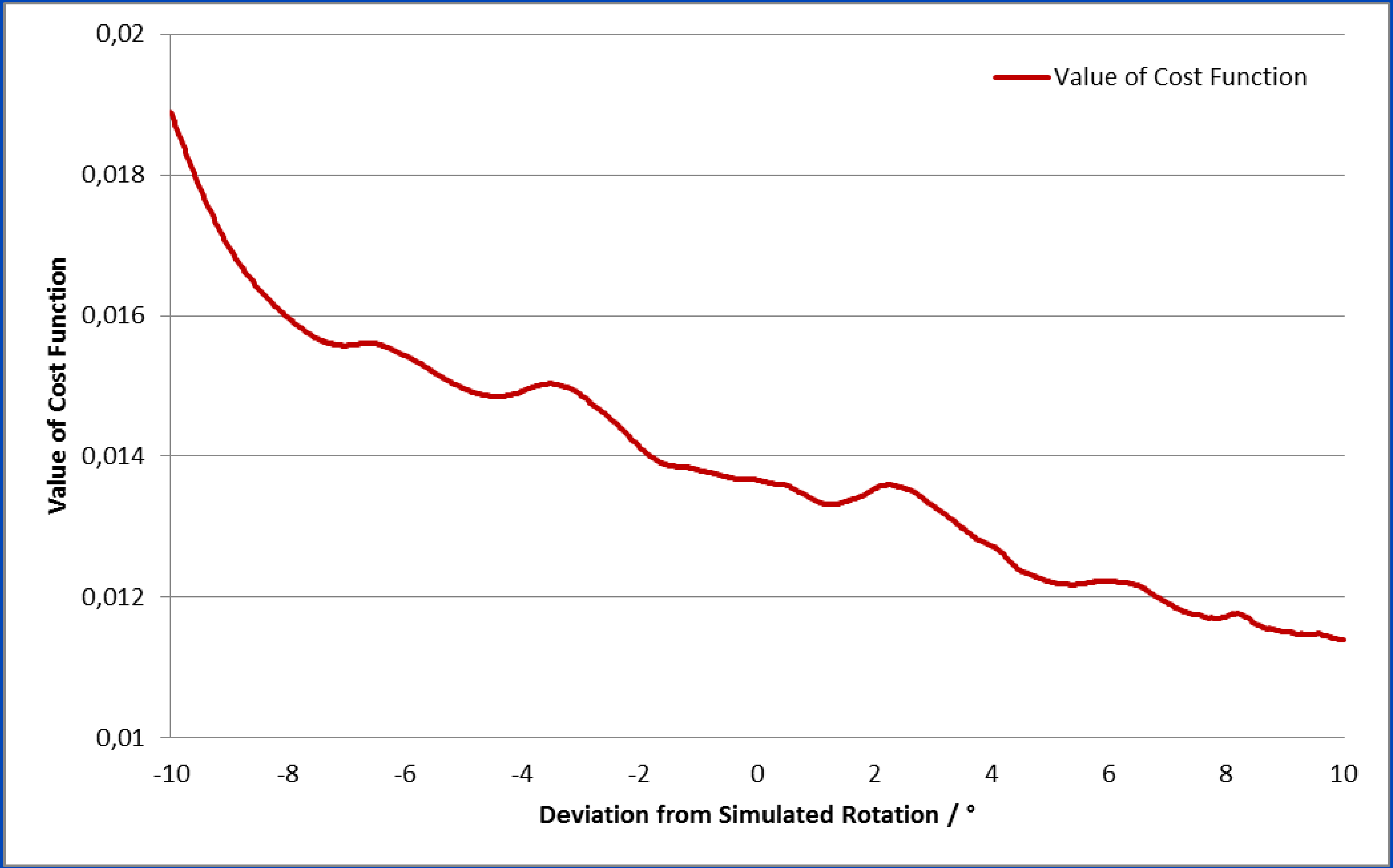
Results: Cost Function

Roll



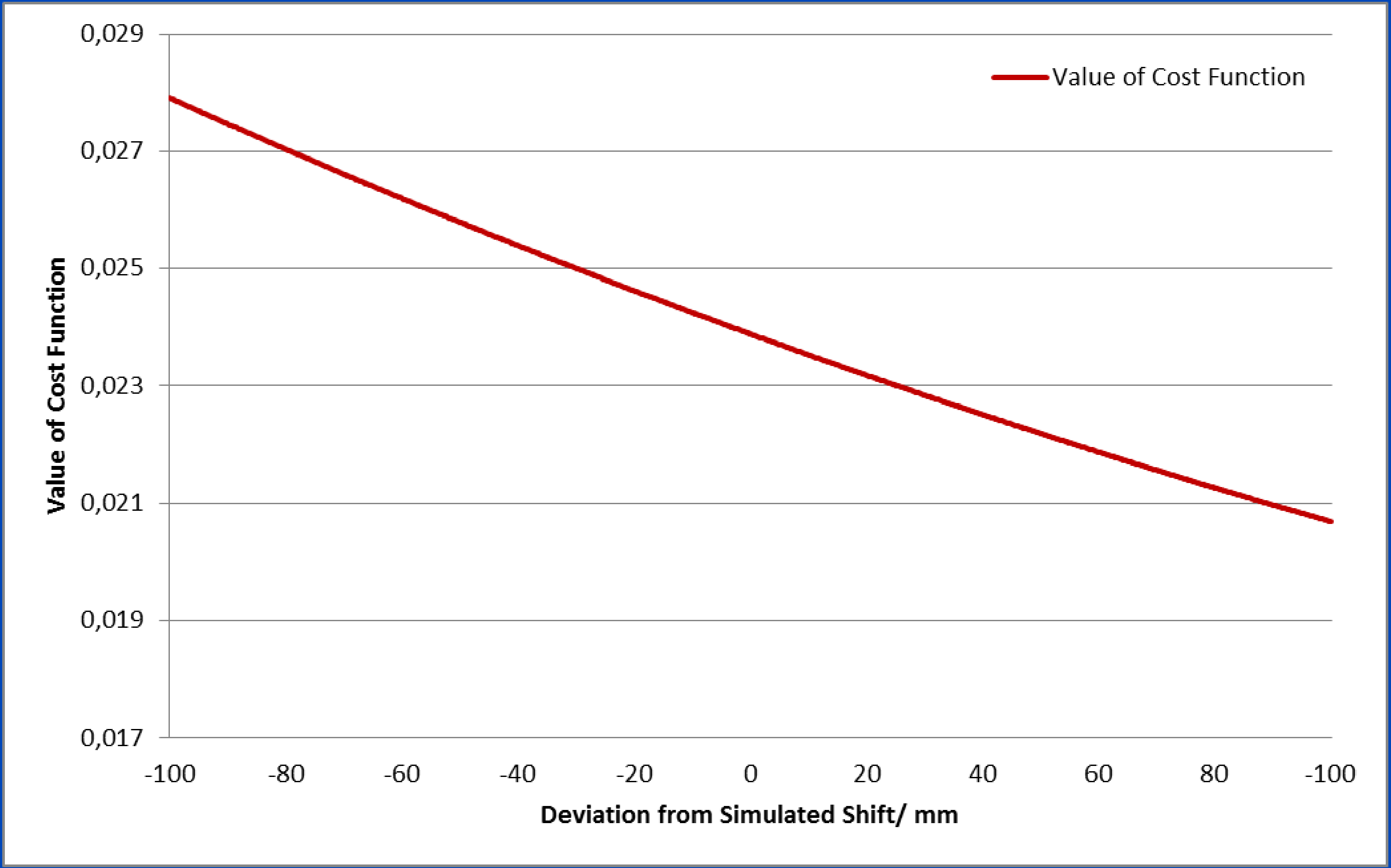
Results: Cost Function

Tilt



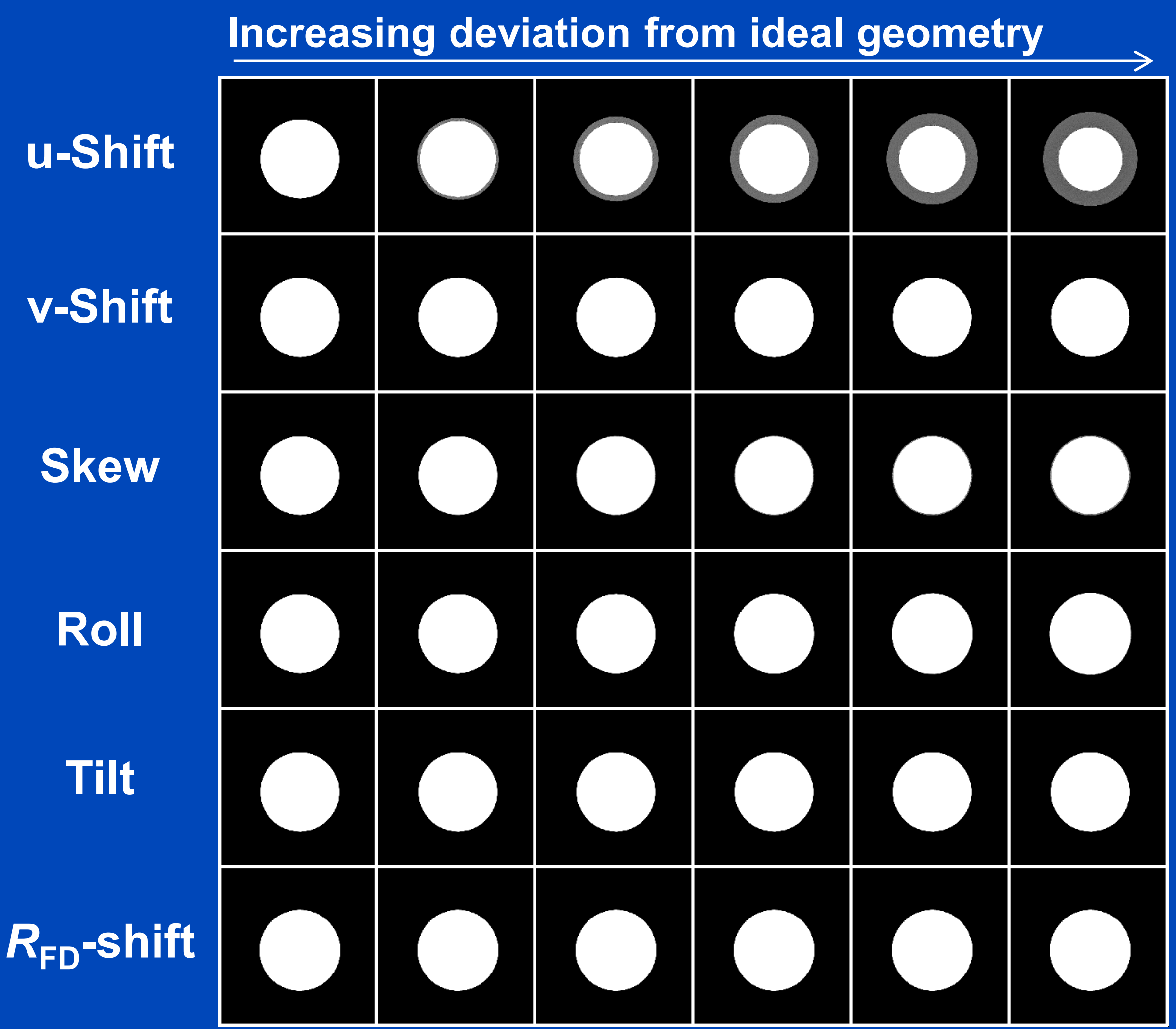
Results: Cost Function

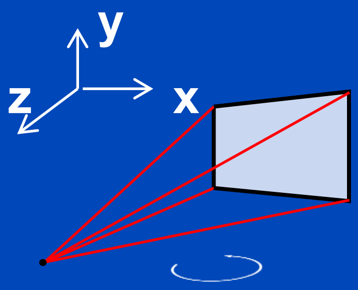
w-Shift (R_{FD}-Shift)



Results: Reconstructions

Uncorrected Reconstructions xy-Slice

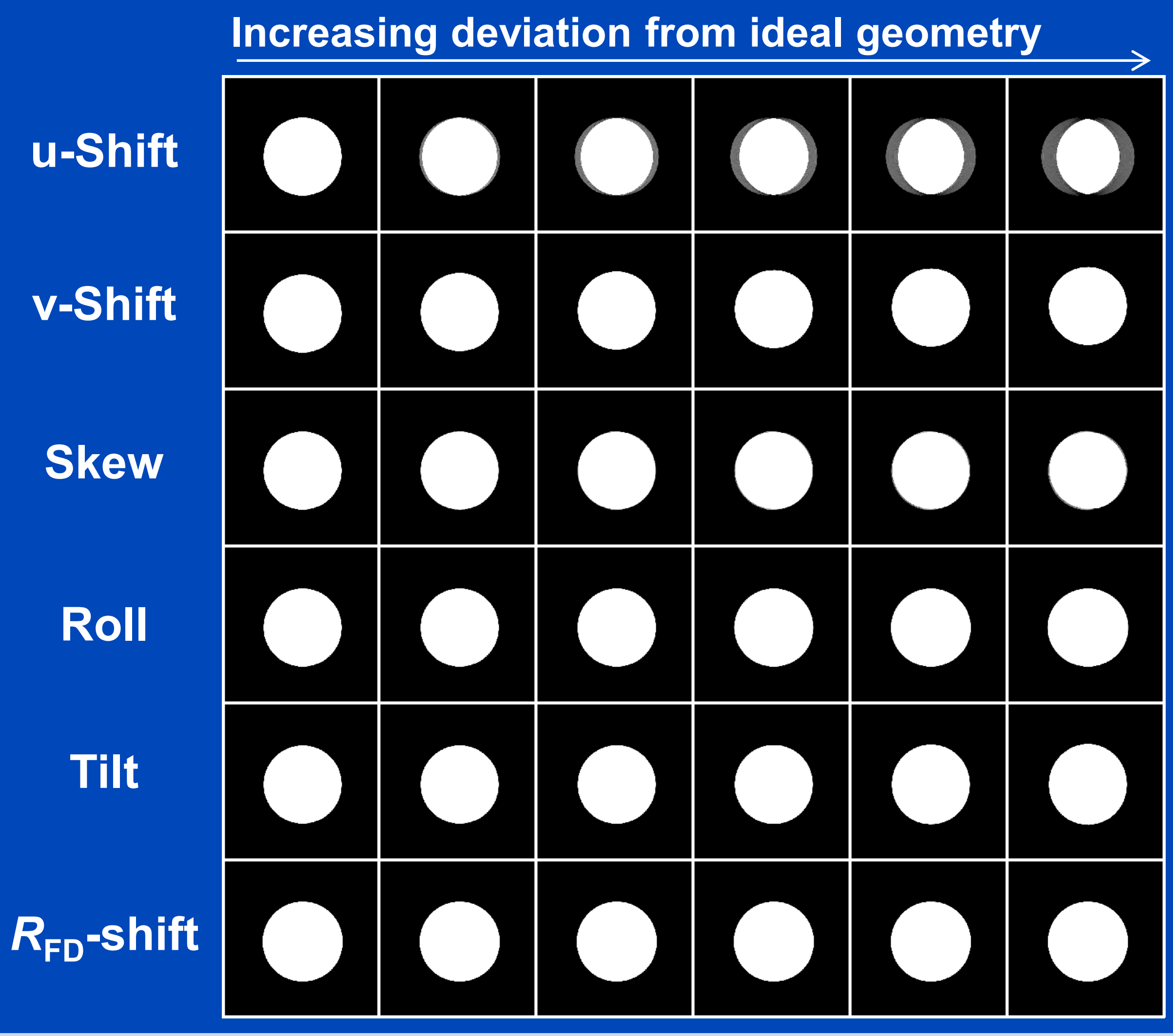


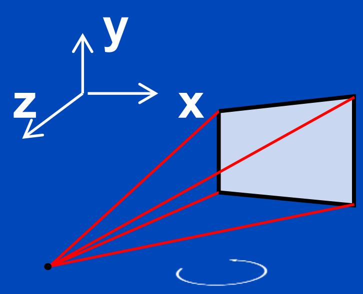


$C / W = (0.5 / 0.5) \cdot 10^{-3}$ / mm

Results: Reconstructions

Uncorrected Reconstructions xz-Slice

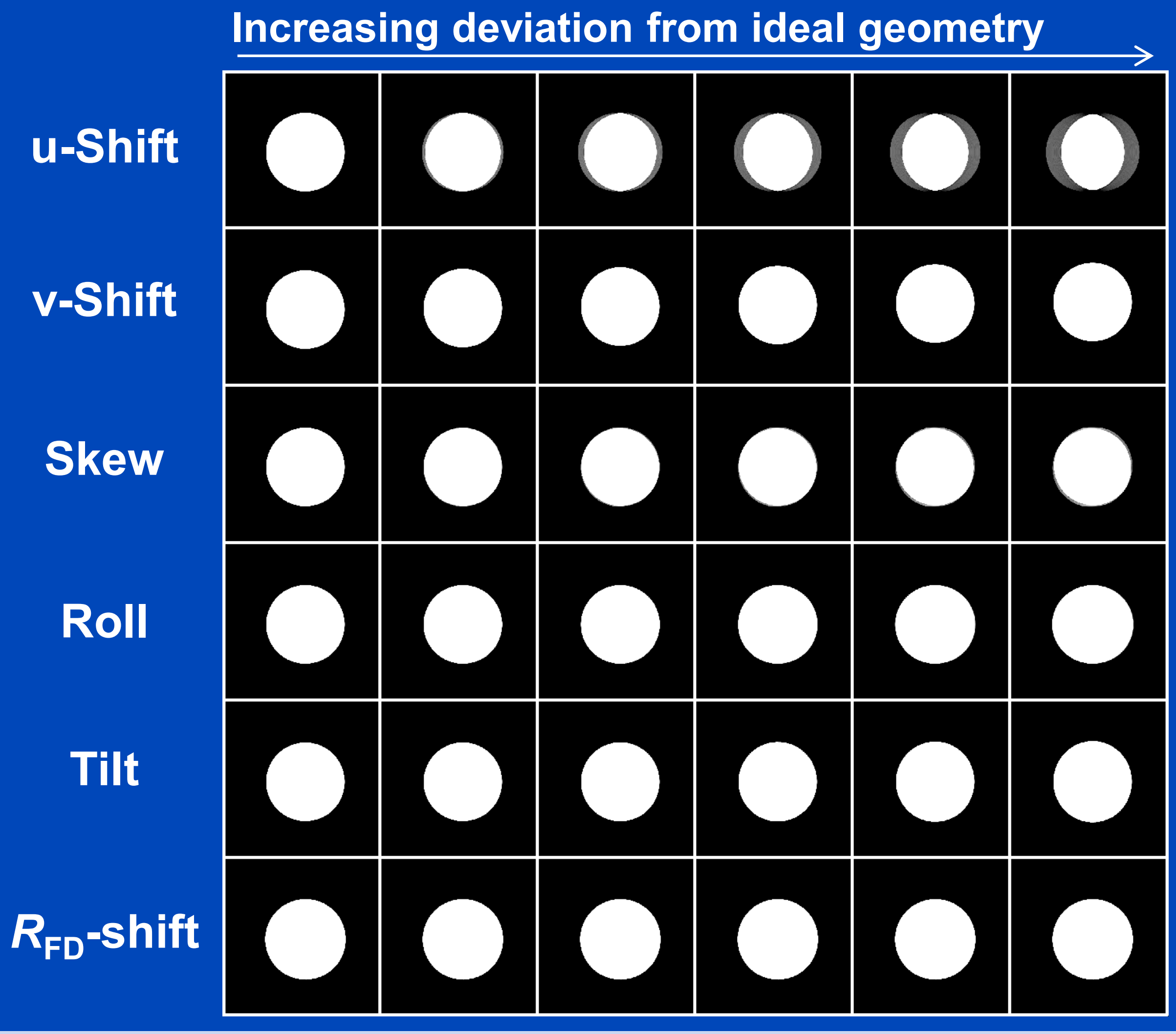


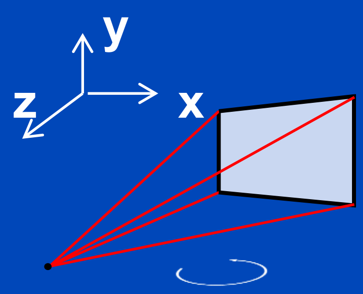


C / W = (0.5 / 0.5) · 10^{-3} / mm

Results: Reconstructions

Uncorrected Reconstructions yz-Slice

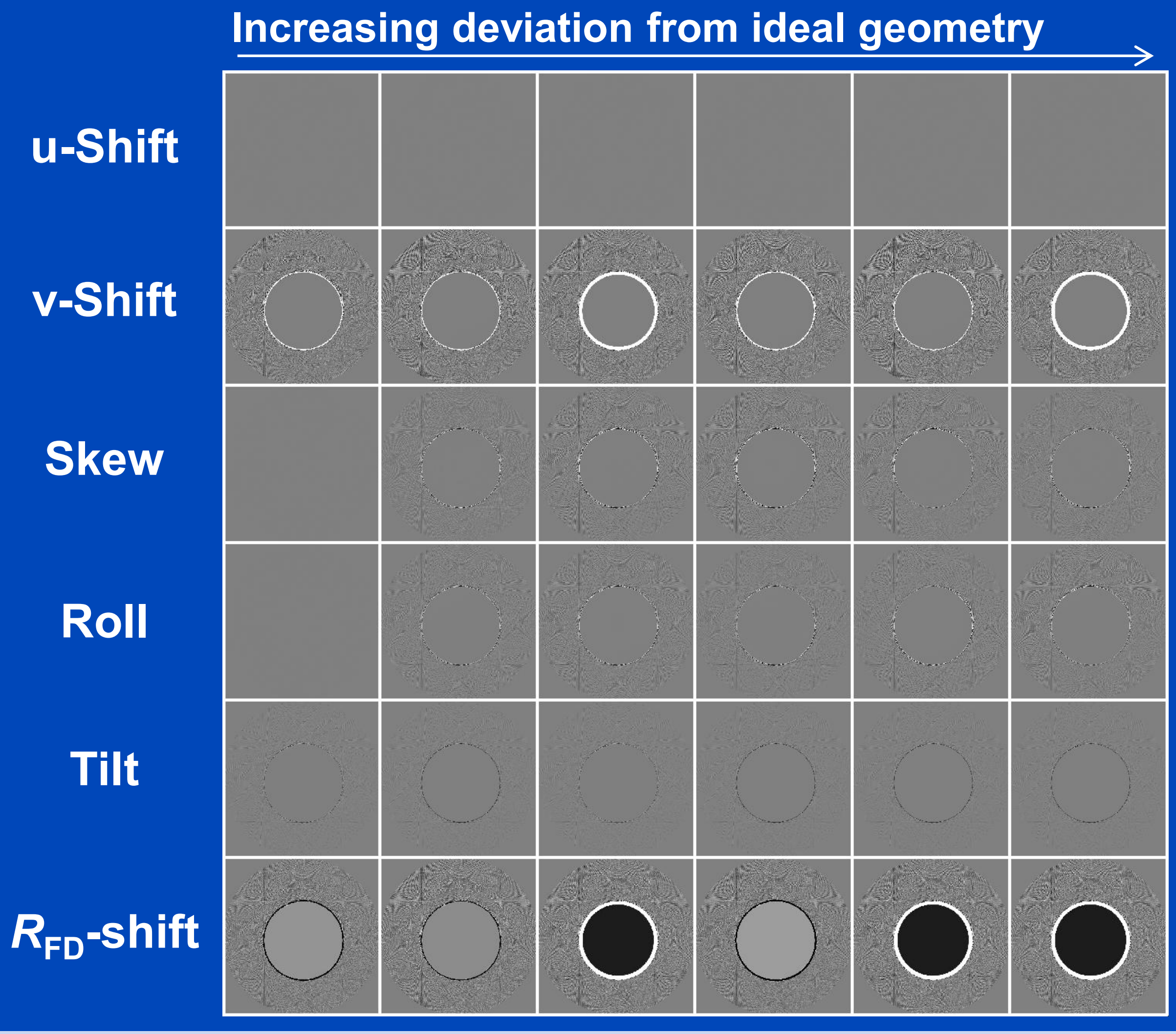


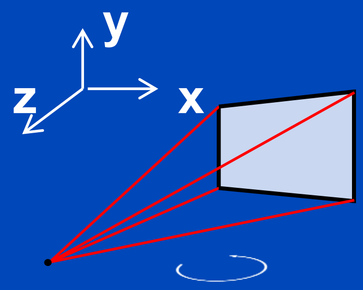


C / W = (0.5 / 0.5) · 10^{-3} / mm

Results: Reconstructions

Difference between Corrected and Ideal Reconstruction xy-Slice



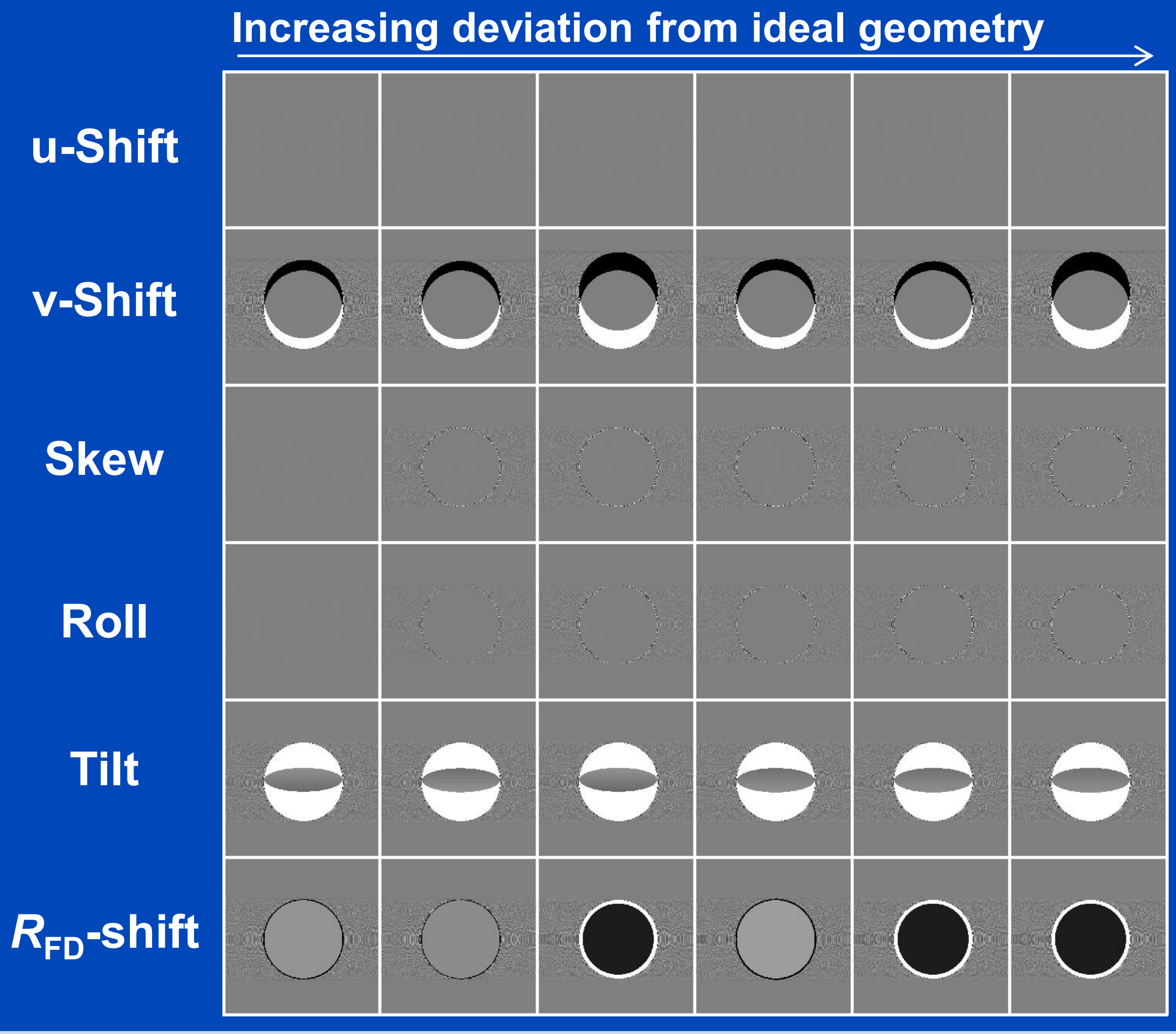


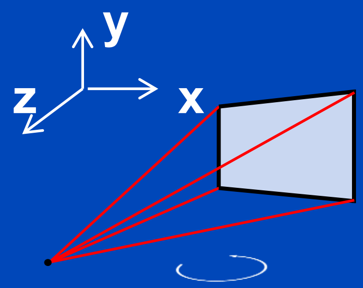
C / W = (0.00 / 0.25) · 10^{-3} / mm

dkfz.

Results: Reconstructions

Difference between Corrected and Ideal Reconstruction xz-Slice

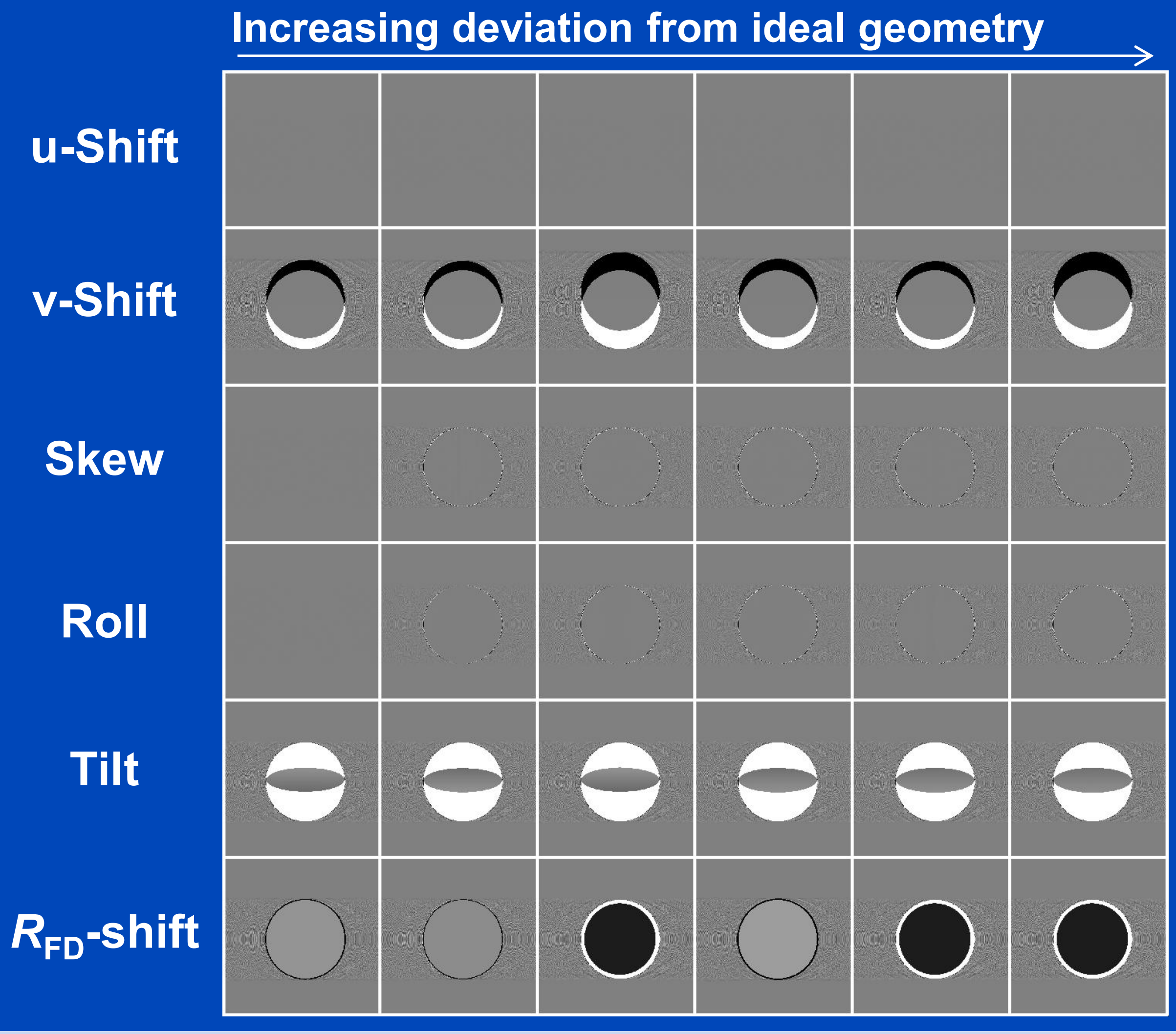


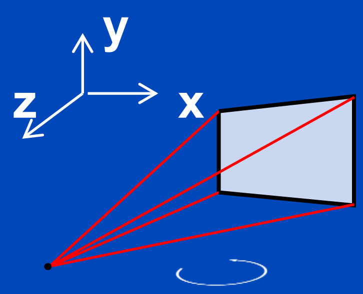


C / W = (0.00 / 0.25) · 10^{-3} / mm

Results: Reconstructions

Difference between Corrected and Ideal Reconstruction yz-Slice





C / W = (0.00 / 0.25) · 10^{-3} / mm

Simulations (Beam Hardening + Misalignment)

- Definition of an ideal geometry
- Simulation of projections of a sphere phantom with different materials (water, bone, aluminum) with a corrupted geometry.
- Deviation of corrupted geometry from ideal geometry
  - U-shift: 50 pixels
  - Skew: 0.08 rad
  - Roll: 0.1 rad
- Simulation of monochromatic data
- For simulation of polychromatic data
  - 60 kV tube voltage, Tucker spectrum
  - No prefilter
  - 0.7 mm energy-integrating CsI detector

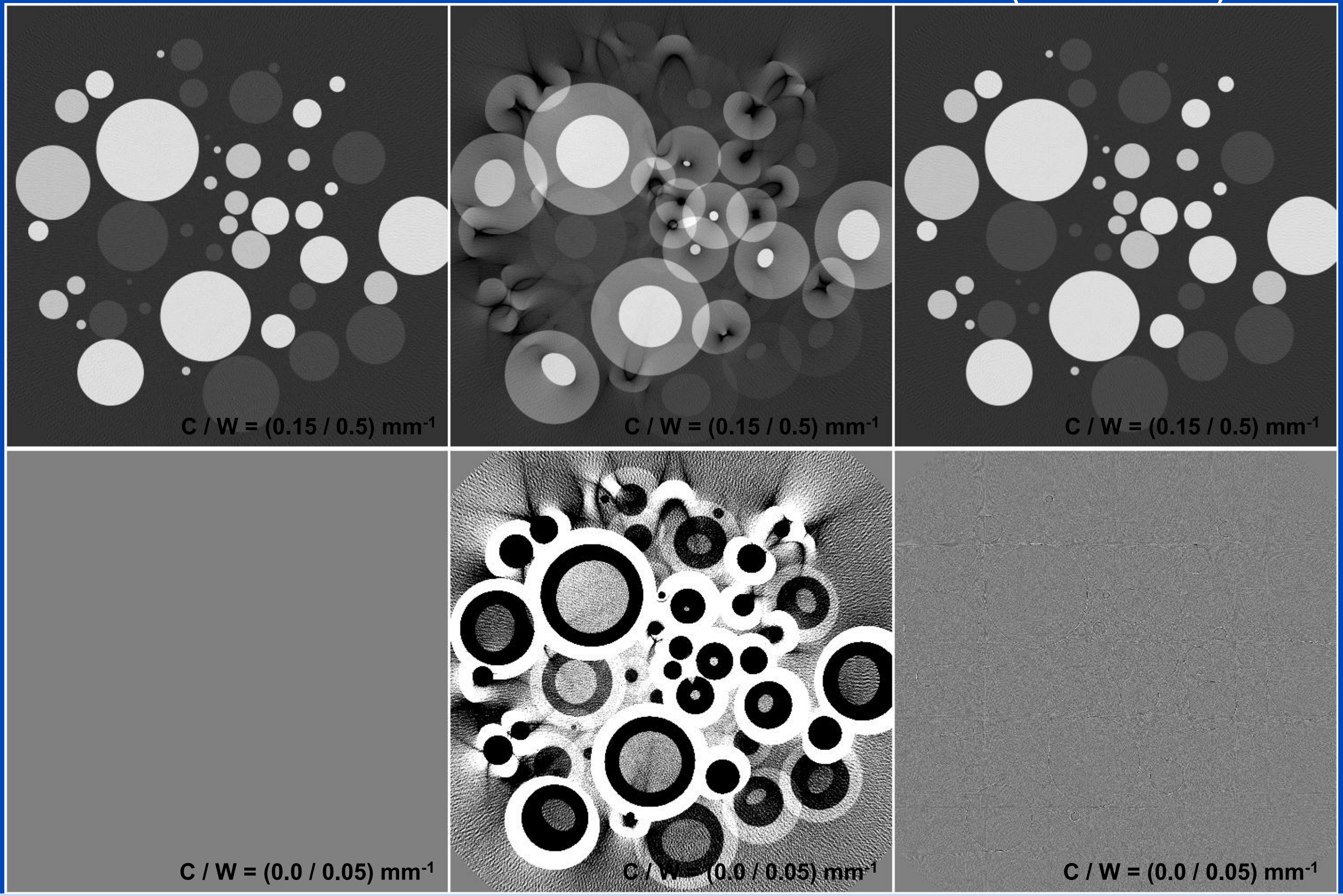
Results

Monochromatic Simulation

Ideal geometry | **Real geometry** | **Estimated geometry (50 / 0.08 / 0.1)**

Reconstructions

Difference to reconstruction with ideal geometry



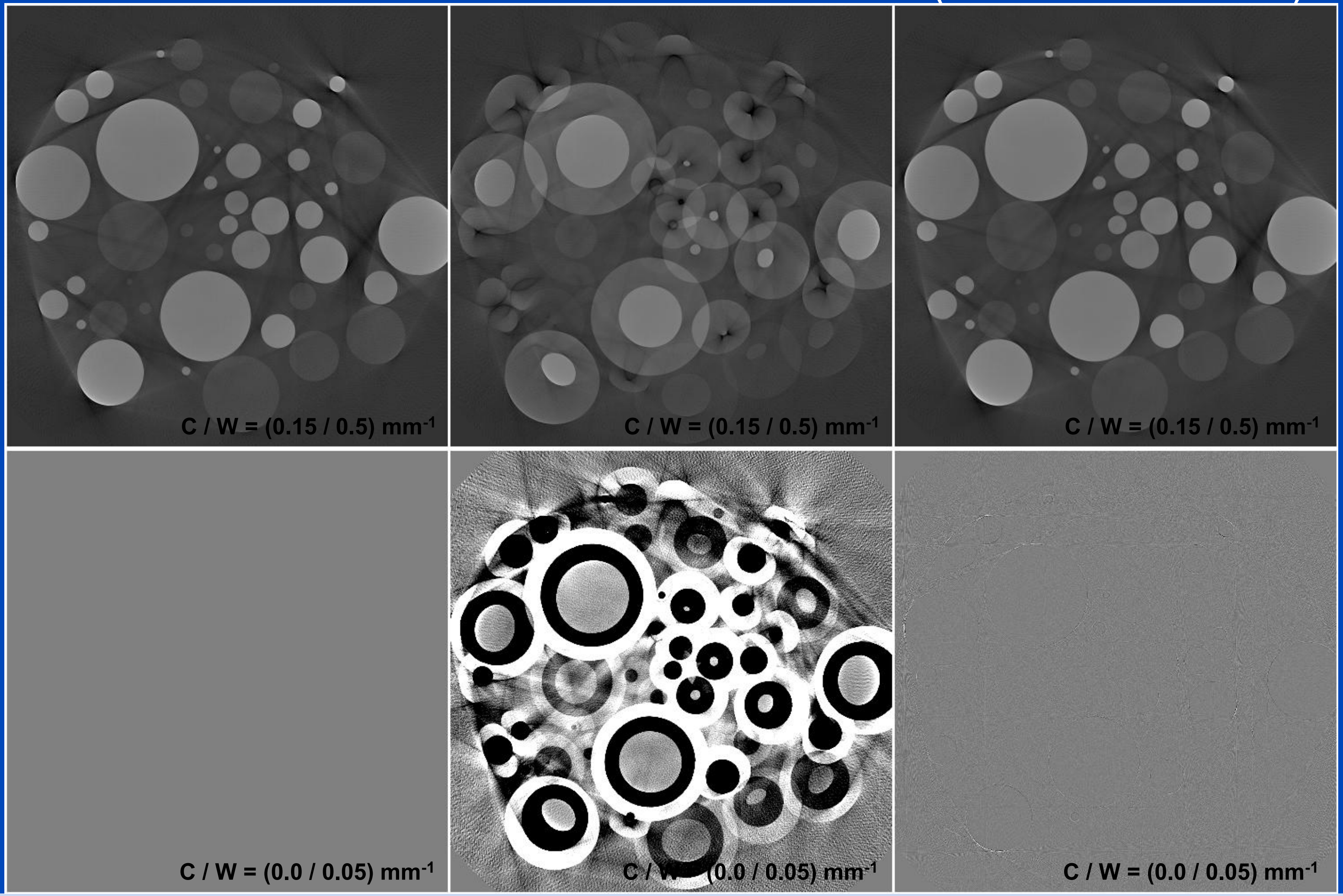
Results

Polychromatic Simulation, No Prefiltration

Ideal geometry | Real geometry | Estimated geometry (49.952 / 0.0799 / 0.0993)

Reconstructions

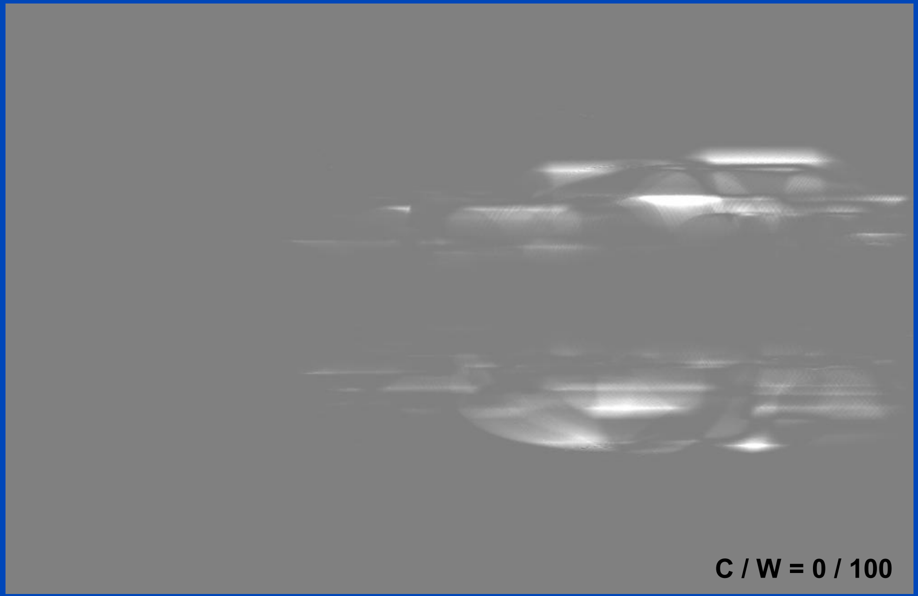
Difference to reconstruction with ideal geometry



Results

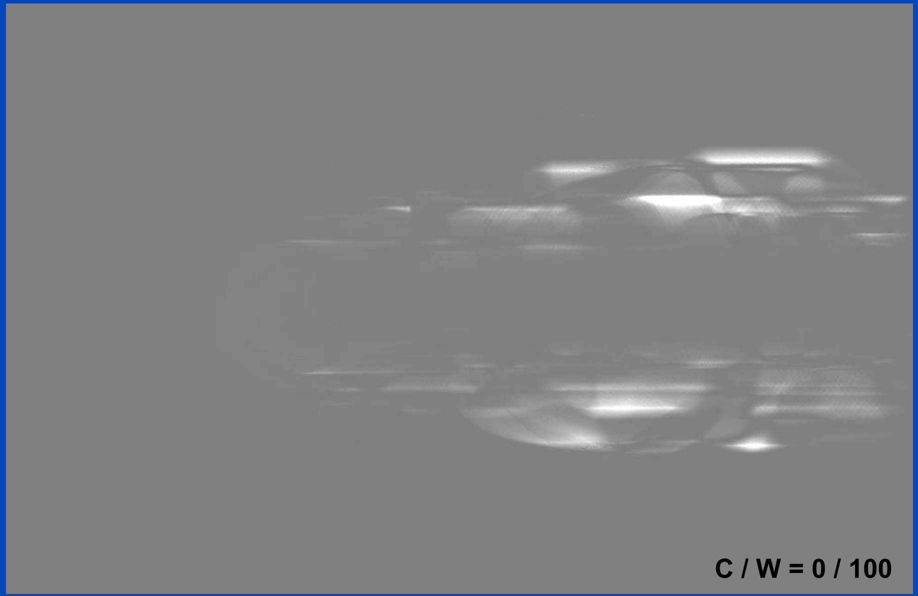
Polychromatic Simulation, No Prefiltration

Squared difference of sum of projections, ideal geometry



Value cost function: 3692146

Squared difference of sum of projections, estimated geometry



Value cost function: 3604906

Simulations (Truncation + Misalignment)

- Definition of an ideal geometry
- Simulation of projections of a sphere phantom with different materials (water, bone, aluminum) with a corrupted geometry.
- Deviation of corrupted geometry from ideal geometry
  - U-shift: 50 pixels
  - Skew: 0.08 rad
  - Roll: 0.1 rad
- Simulation of monochromatic data
- Simulation of truncation in u- and v-direction by setting projection values at the periphery to zero

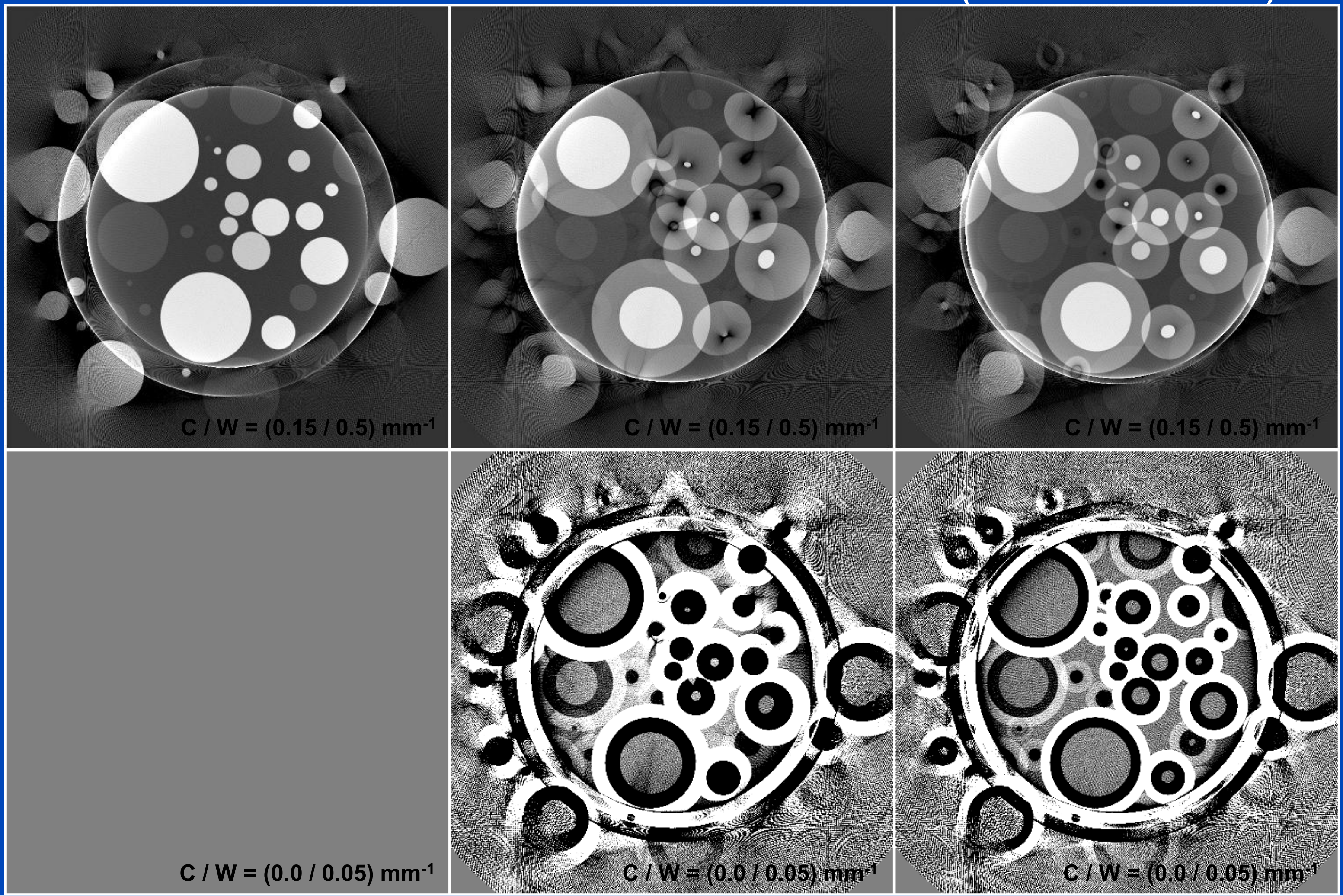
Results

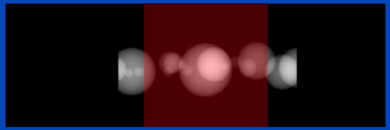
Monochromatic Simulation, Truncation in u-direction,

Ideal geometry | Real geometry | Estimated geometry (15.32 / 0.082 / 0.150)

Reconstructions

Difference to reconstruction with ideal geometry





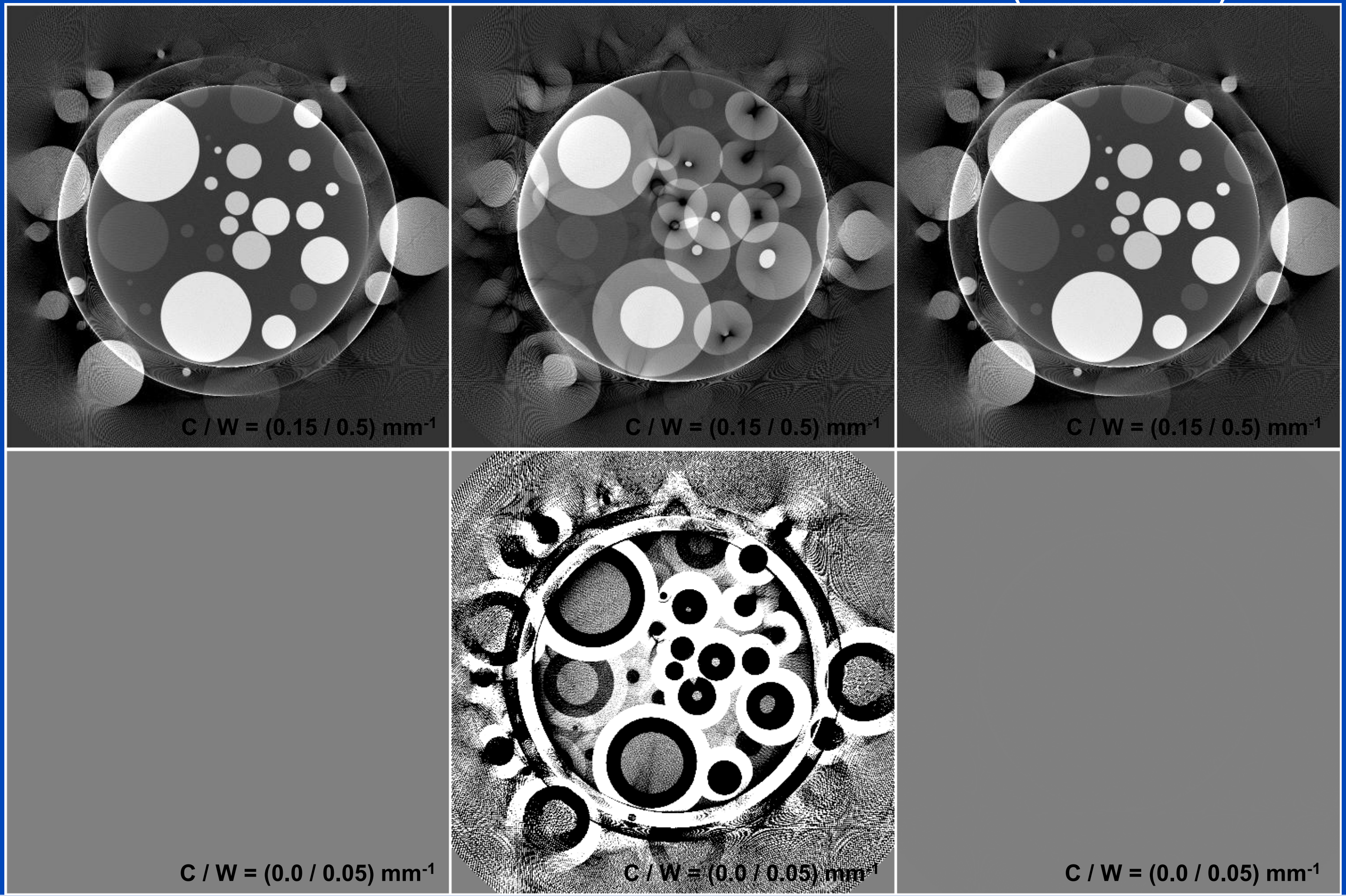
Results

Monochromatic Simulation, Truncation in u-direction, limited area of optimization

Ideal geometry | Real geometry | Estimated geometry (50 / 0.08 / 0.1)

Reconstructions

Difference to reconstruction with ideal geometry



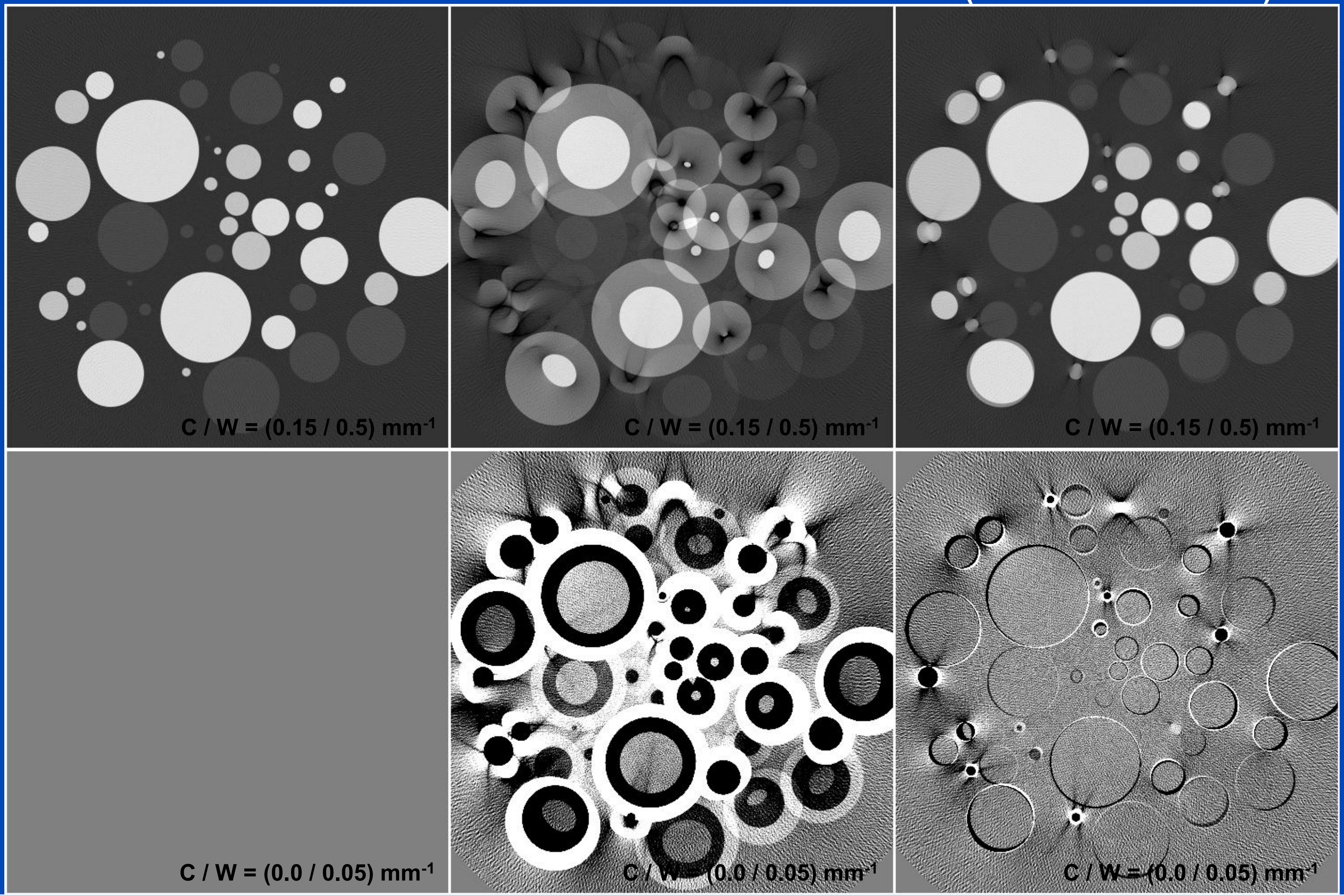
Results

Monochromatic Simulation, Truncation in v-direction,

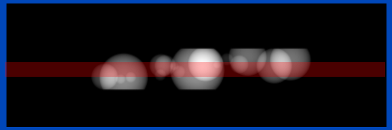
Ideal geometry | Real geometry | Estimated geometry (55.94 / 0.047 / -0.15)

Reconstructions

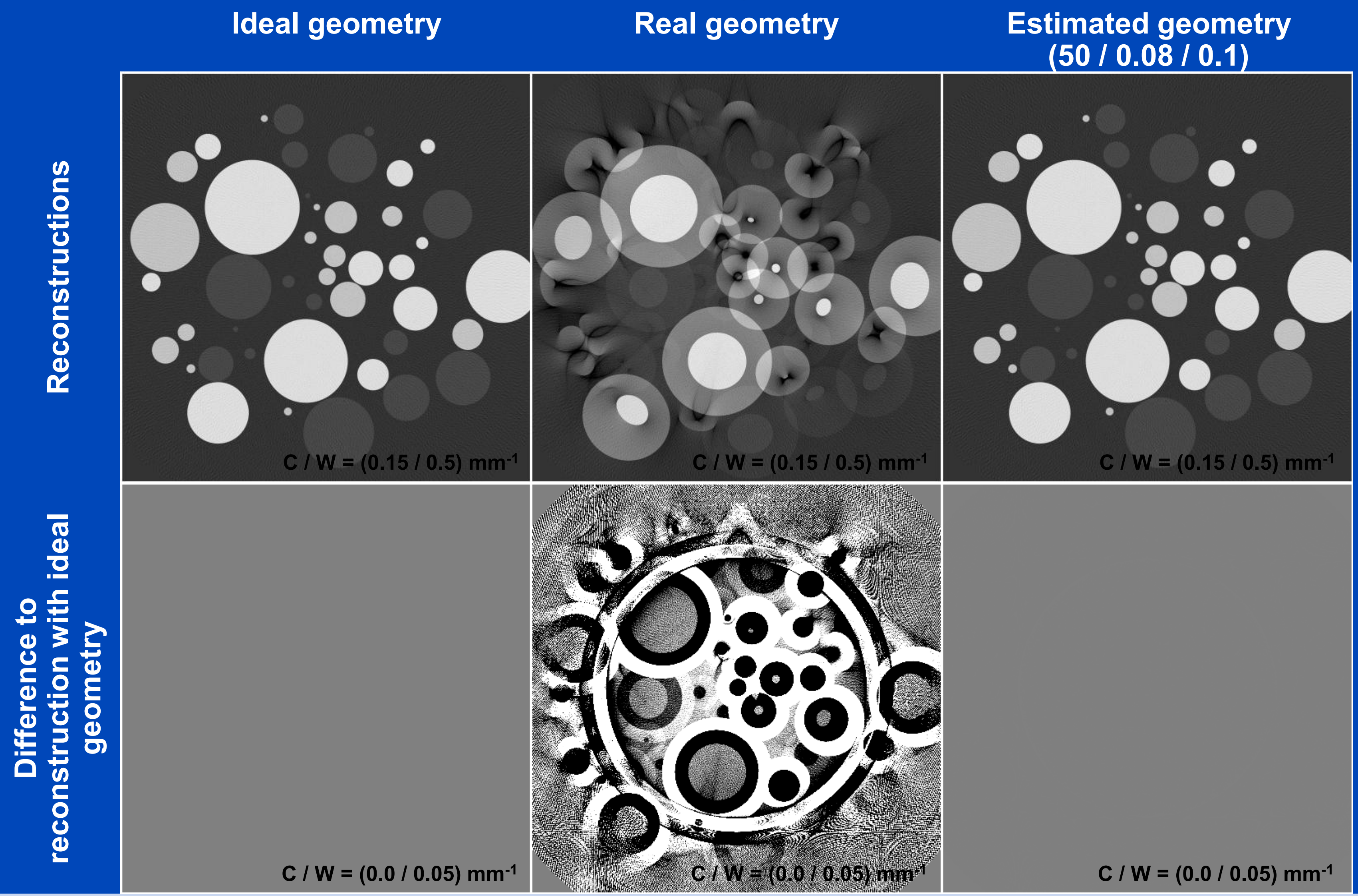
Difference to reconstruction with ideal geometry



Results



Monochromatic Simulation, Truncation in v-direction, limited area of optimization



Conclusions

- The method of Meng et al. is able to estimate u-shift, skew and roll.
- For u-shift, skew and roll the cost function has a well-defined minimum.
- The parameters v-shift, tilt and R_{FD}-shift cannot be well estimated.
- The periodic behavior of the cost function for a variation of v-shift might result from the interpolation process of the performed rebinning.
- Beam hardening seems to have only a very small influence on the performance of the estimation.
- Truncation influences the performance of the estimation if it is not considered appropriately within the optimization process.

Thank You!

- This presentation will soon be available at www.dkfz.de/ct.
- Job opportunities through DKFZ's international PhD or Postdoctoral Fellowship programs (marc.kachelriess@dkfz.de).
- Parts of the reconstruction software were provided by RayConStruct® GmbH, Nürnberg, Germany.