Detector and Noise

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Offset and Gain Calibration

- Acquire many offset (or dark) images $O(u, v)$ and average them.
- Acquire many gain (or bright) images $G(u, v)$ and average them.
- Basic preprocessing of actual x-ray image $I(u, v)$
  - subtract offset
  - divide by gain minus offset
  - rescale

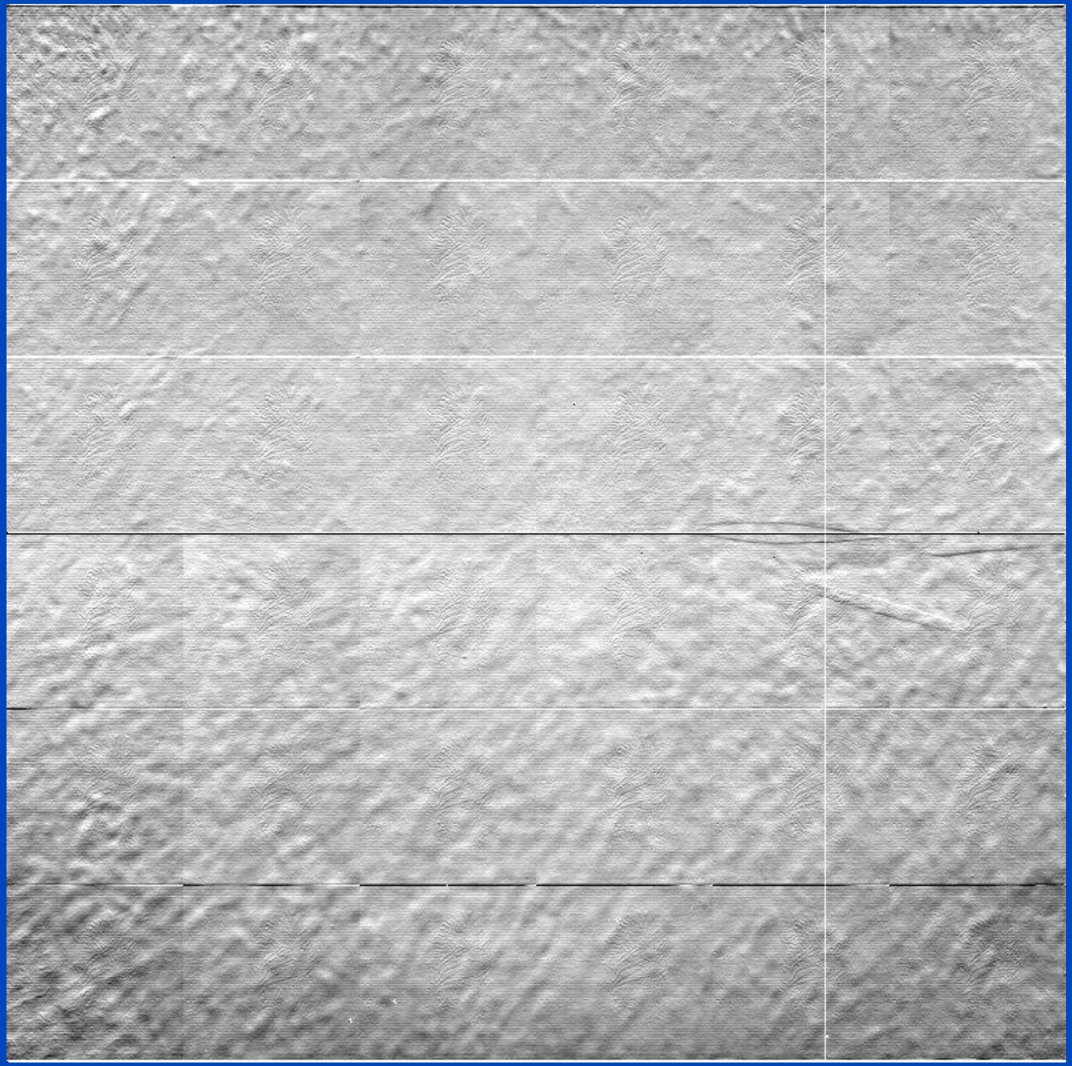
$$I(u,v) = \frac{I(u,v) - O(u,v)}{G(u,v) - O(u,v)} \langle G(u,v) - O(u,v) \rangle_{u,v}$$

Precorrection Depends on Detector Modes

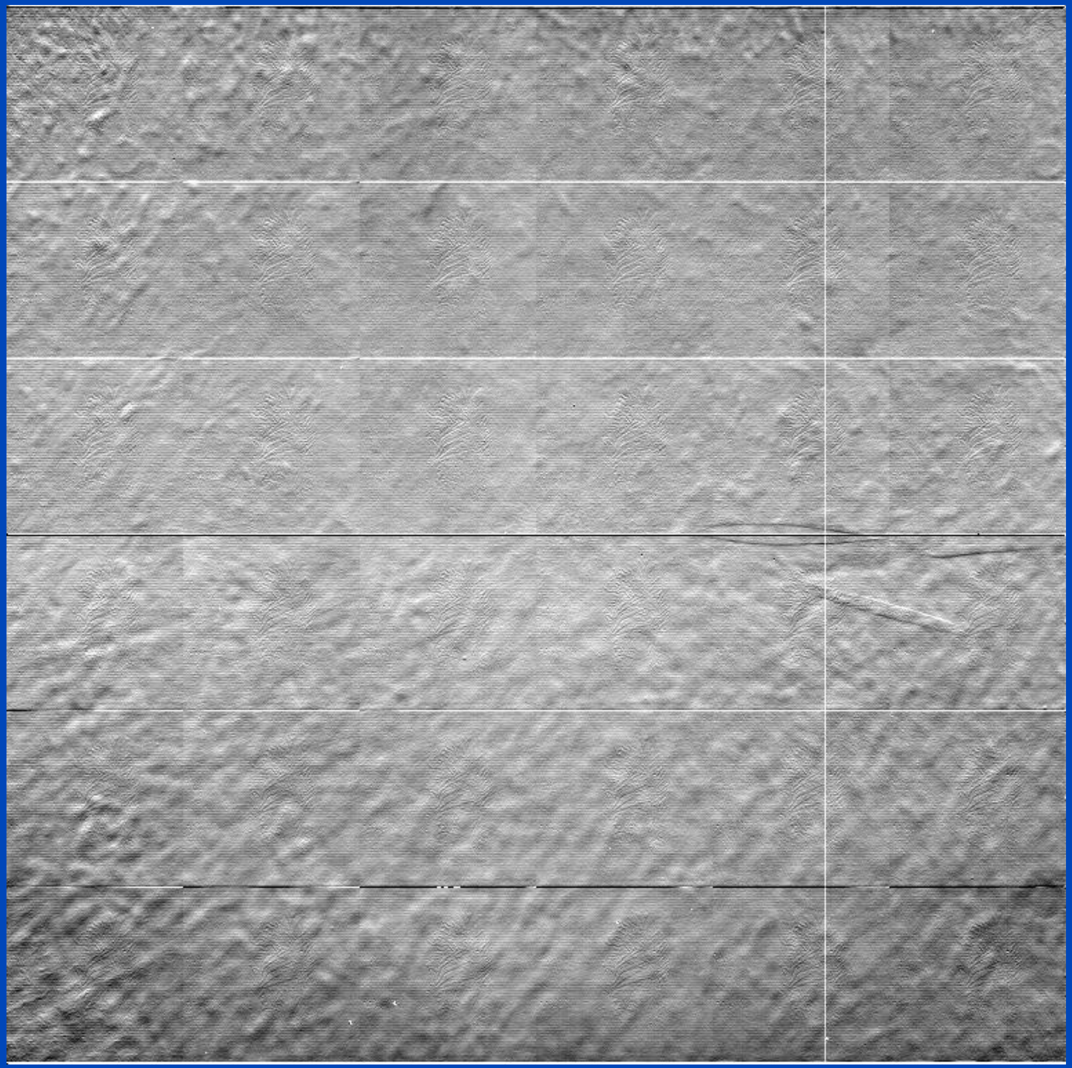
E.g. Varian PaxScan3030+

- 1×1 hardware binning, 0.5 pF, gain 5, 25 fps (mode M0)
- 2×2 hardware binning, 0.5 pF, gain 2, 25 fps (mode M1)
- 1×1 hardware binning, 0.5 pF, gain 5, 12.5 fps (mode M2)
- 2×2 hardware binning, 4.0 pF, gain 2, 25 fps (mode M3)

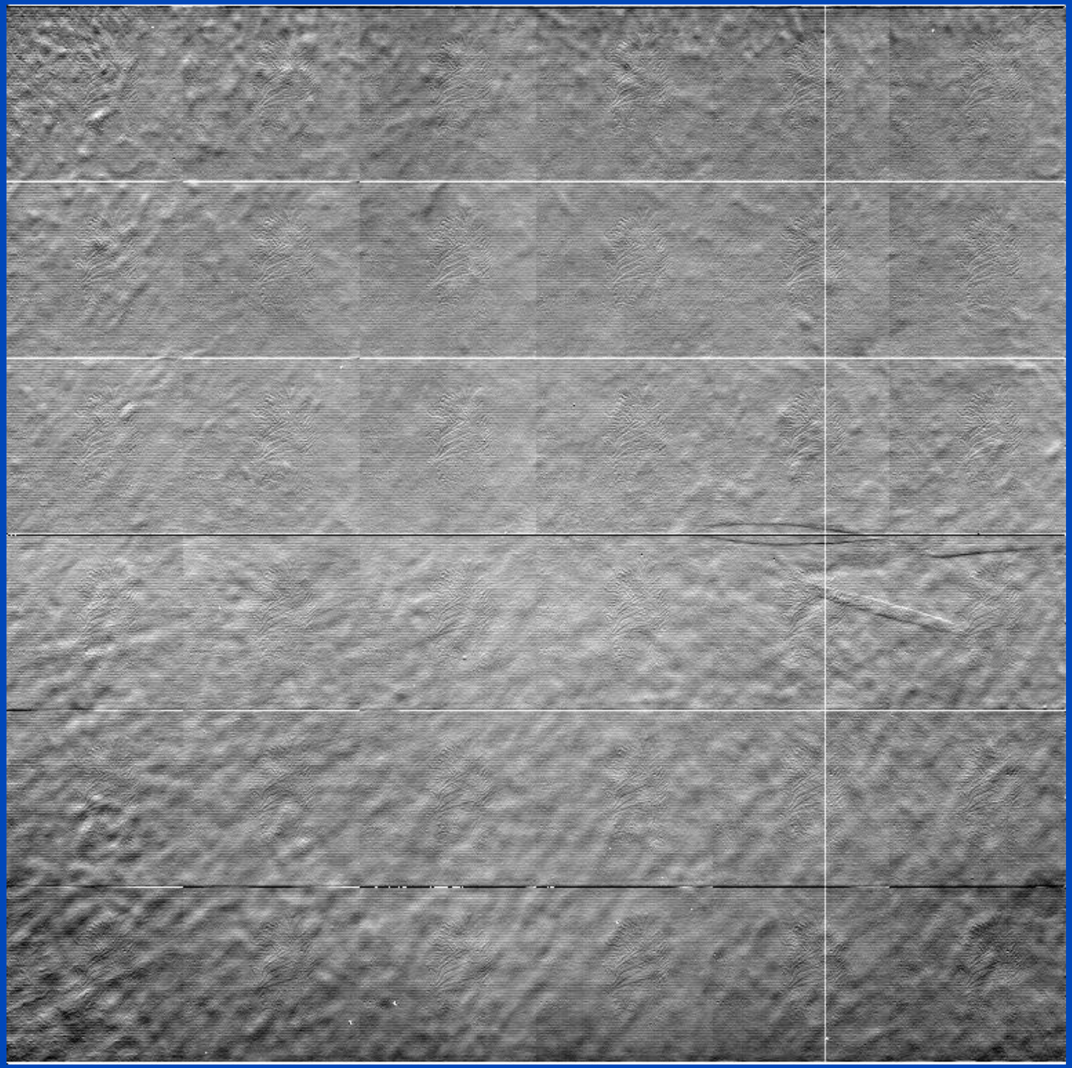
Offset Image 0 min



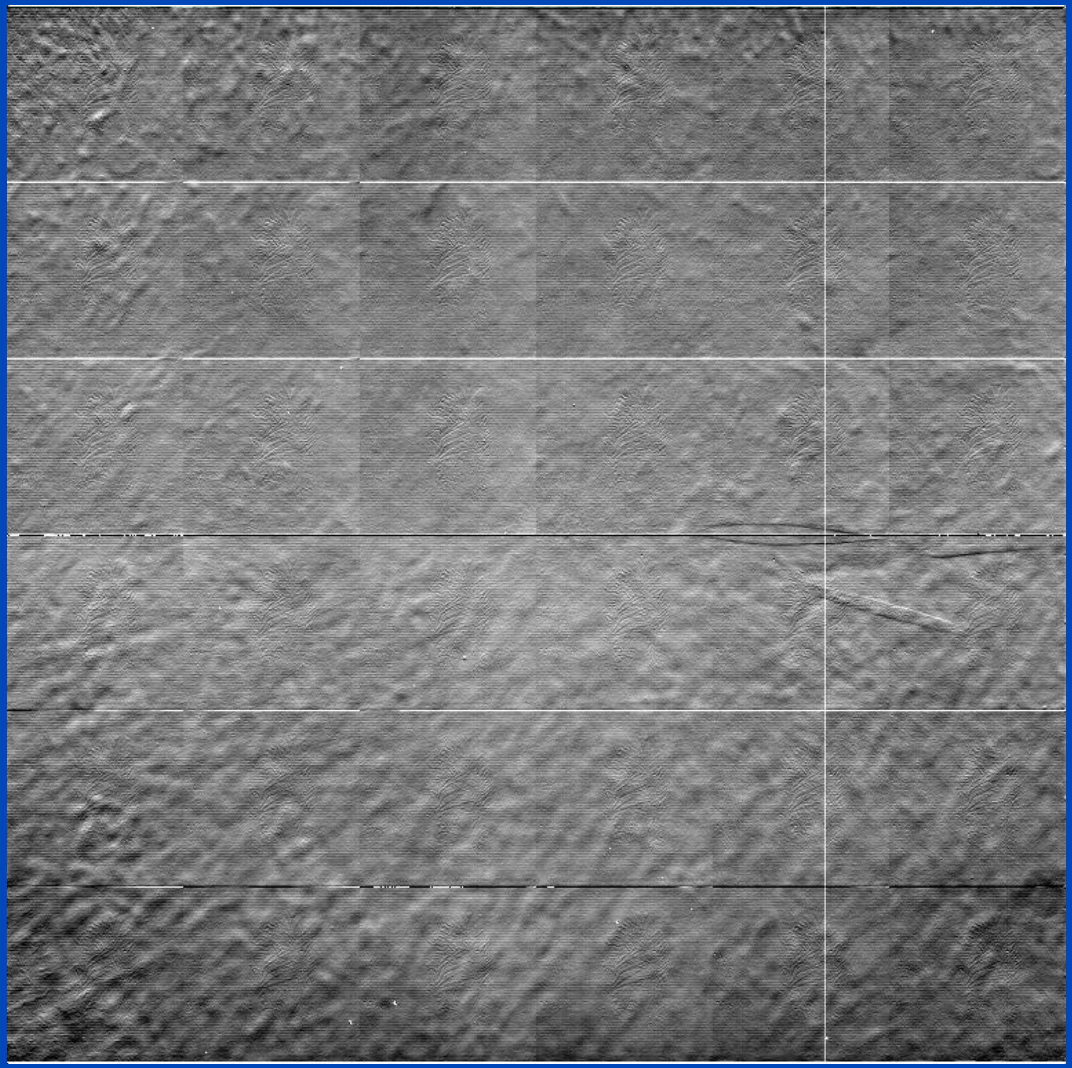
Offset Image 15 min



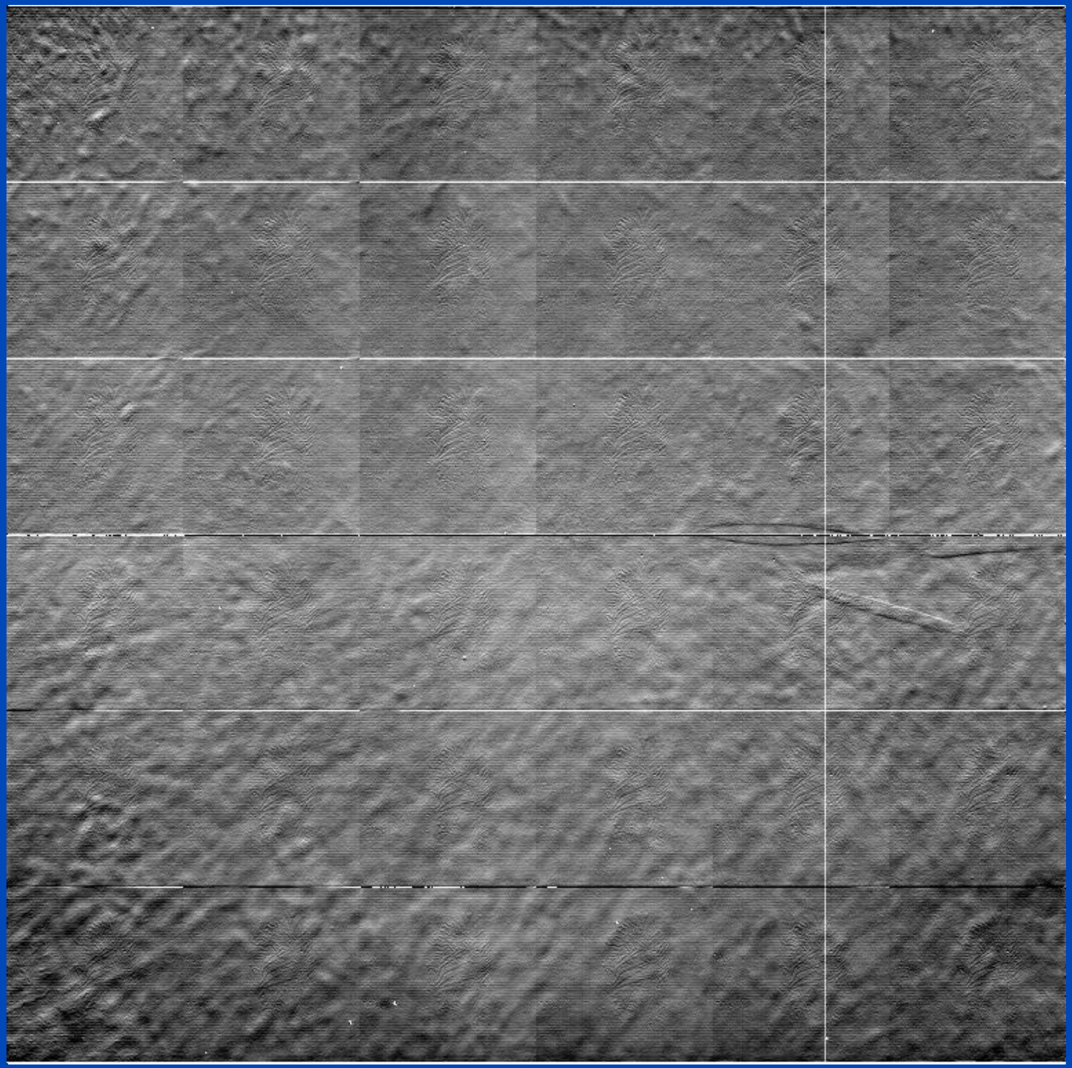
Offset Image 30 min



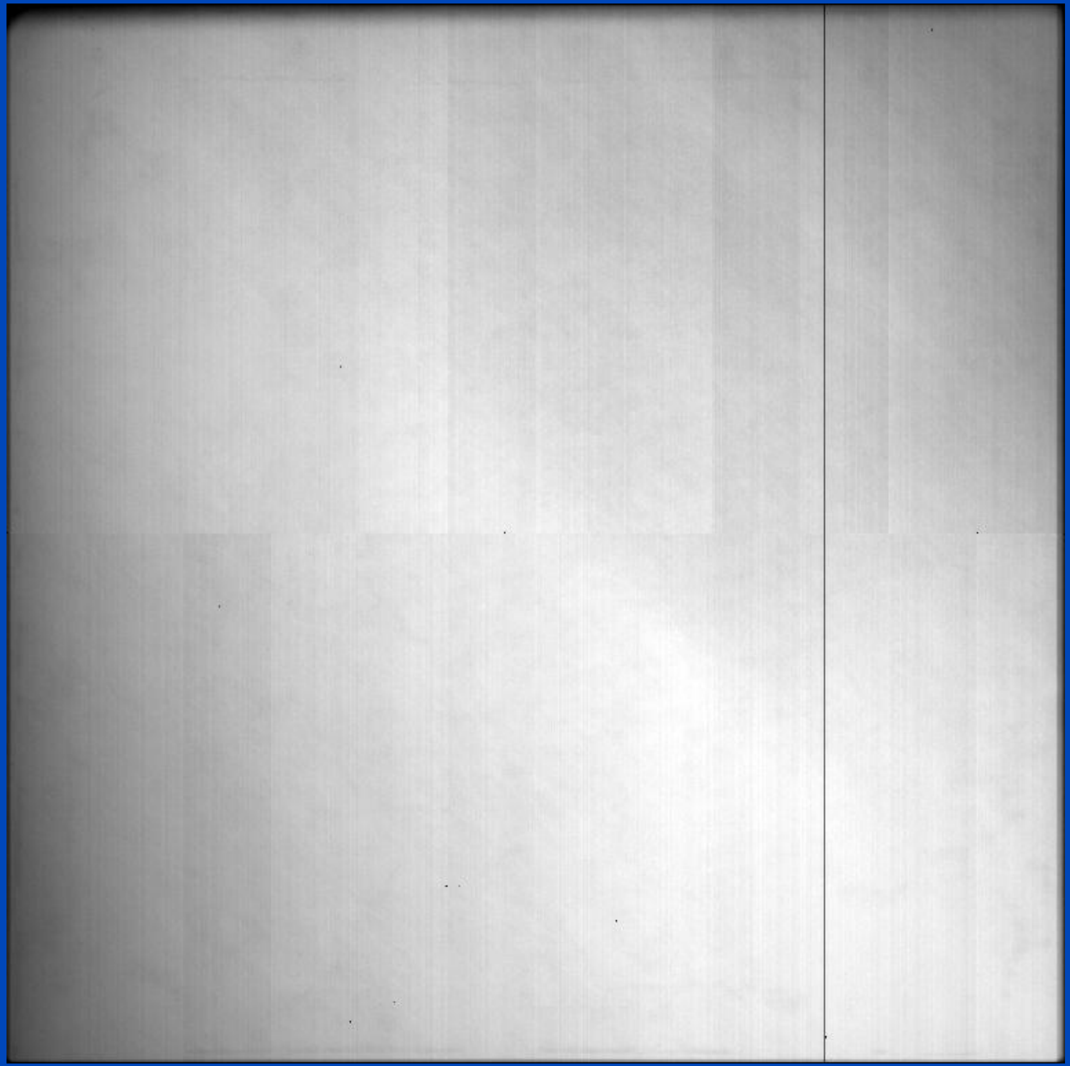
Offset Image 45 min



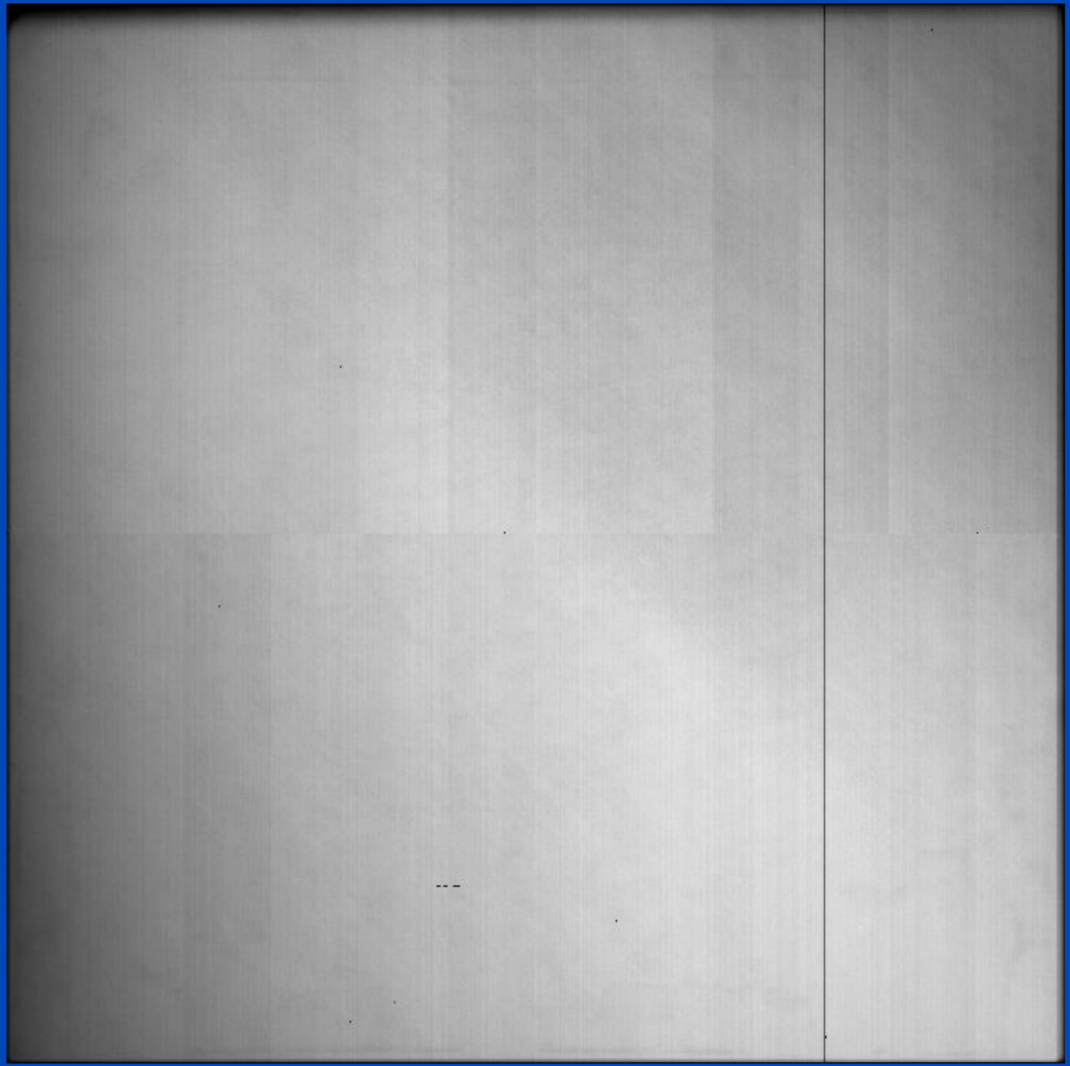
Offset Image 60 min



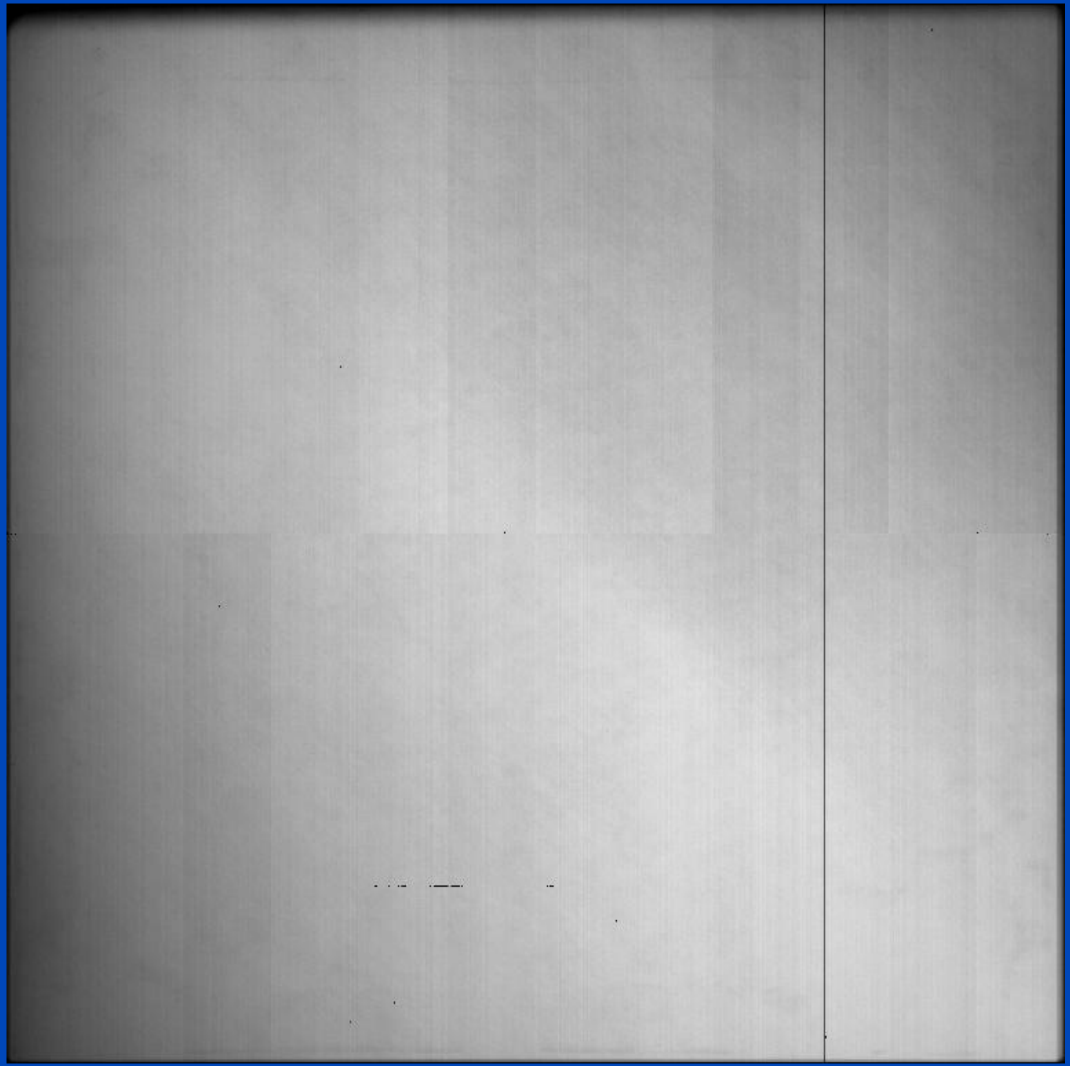
Gain Minus Offset 0 min



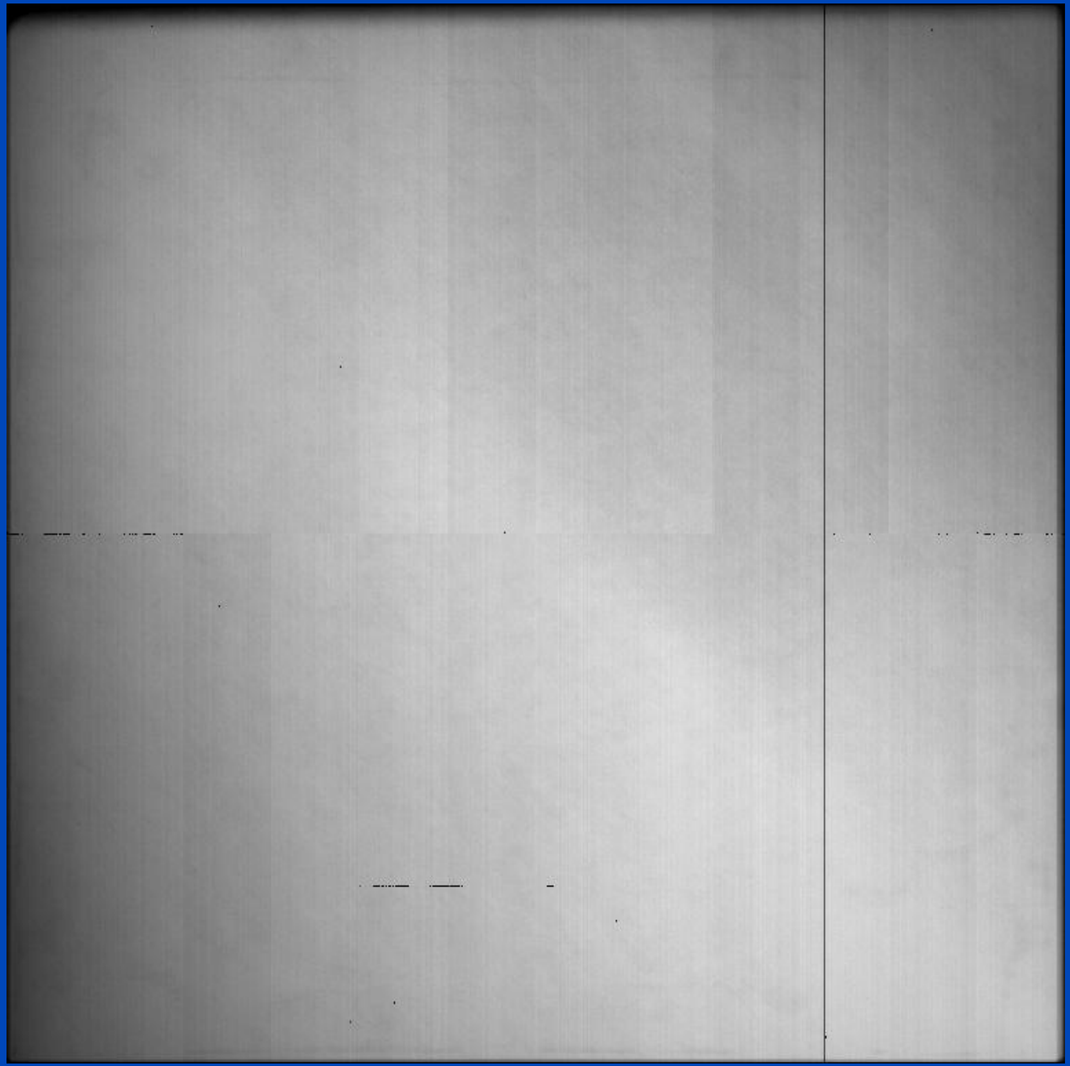
Gain Minus Offset 15 min



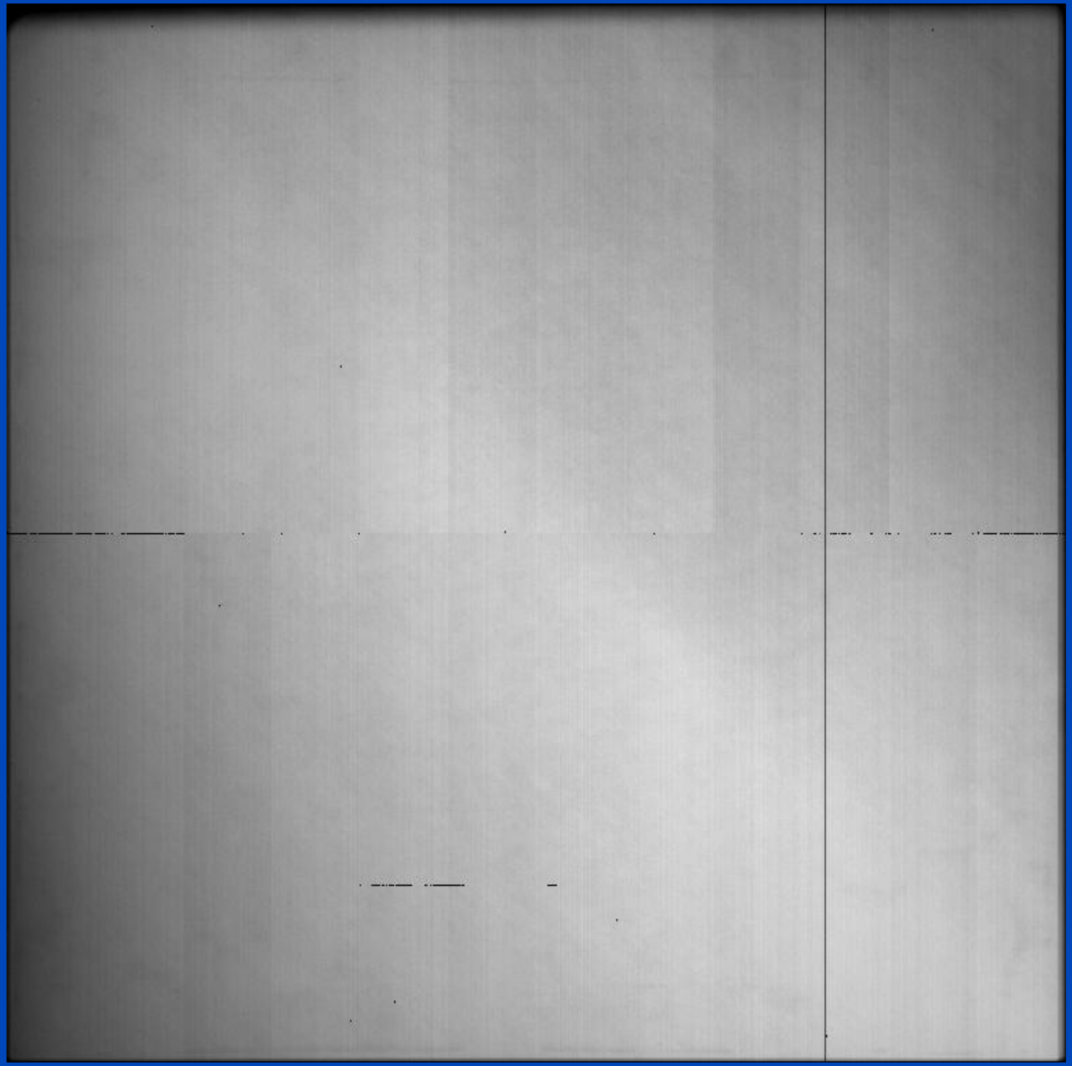
Gain Minus Offset 30 min



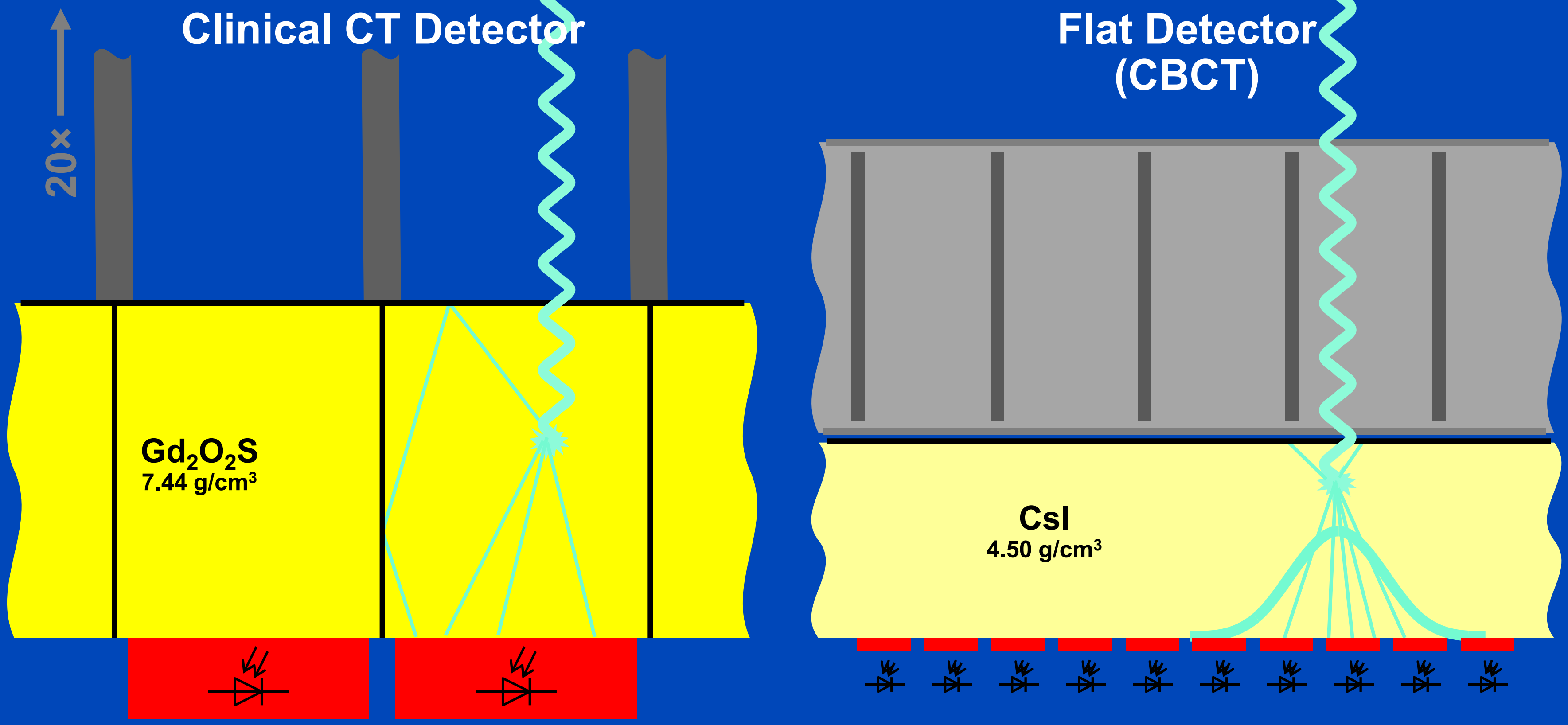
Gain Minus Offset 45 min



Gain Minus Offset 60 min



SPATIAL RESOLUTION AND BINNING



- Anti-scatter grids are aligned to the detector pixels
- Anti-scatter grids reject scattered radiation
- Detector pixels are of about 1 mm size
- Detector pixels are structured, reflective coating maximizes light usage and minimizes cross-talk
- Thick scintillators improve dose usage
- Gd_2O_2S is a high density scintillator with favourable decay times
- Individual electronics, fast read-out (5 kHz)
- Very high dynamic range (10^7) can be realized

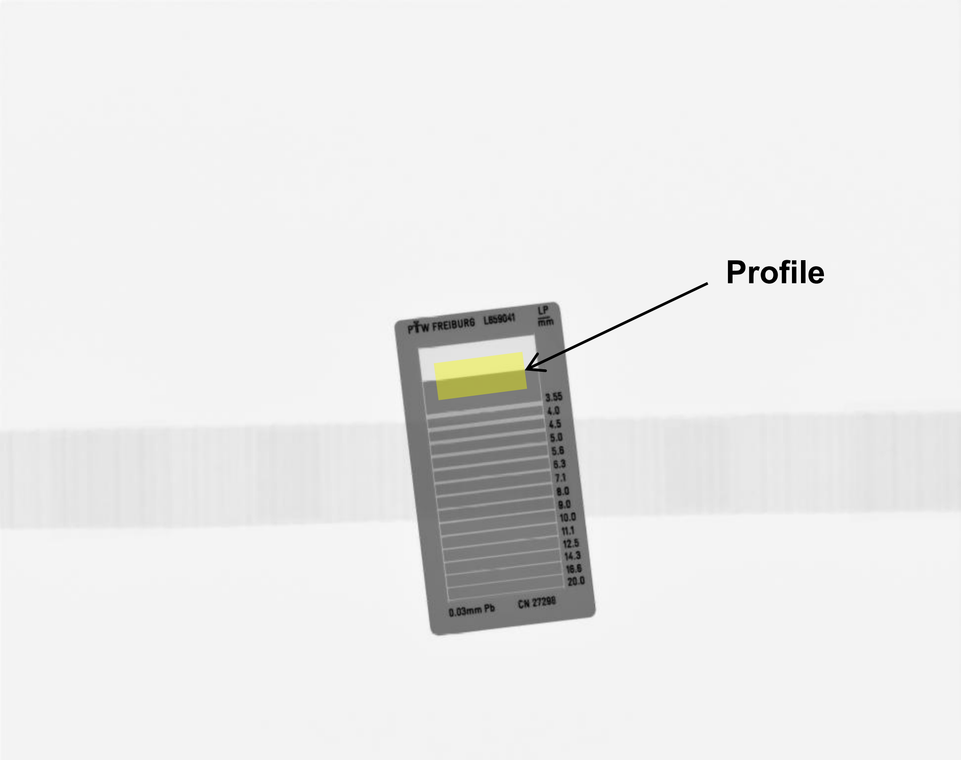
- Anti-scatter grids are not aligned to the detector pixels
- The benefit of anti-scatter grids is unclear
- Detector pixels are of about 0.2 mm size
- Detector pixels are unstructured, light scatters to neighboring pixels, there is significant cross-talk
- Thick scintillators decrease spatial resolution
- CsI grows columnar and suppresses light scatter to some extent
- Row-wise readout is rather slow (e.g. 25 Hz)
- Low dynamic range (<10^3), long read-out paths

Aim

- In conventional energy-integrating detectors x-rays are converted to optical photons using a scintillator crystal.
- Since optical photons are emitted isotropically, the detector line spread function (LSF) broadens if the scintillator thickness is increased.
- Measurement of the detector LSF for two Perkin Elmer Dexela2923 detectors with two scintillator thicknesses, 150 µm and 600 µm.
- Measurement with the following binning settings:
  - 1x1 binning (74.8 µm pixel size)
  - 2x2 binning (149.6 µm pixel size)
  - 4x4 binning (299.2 µm pixel size)

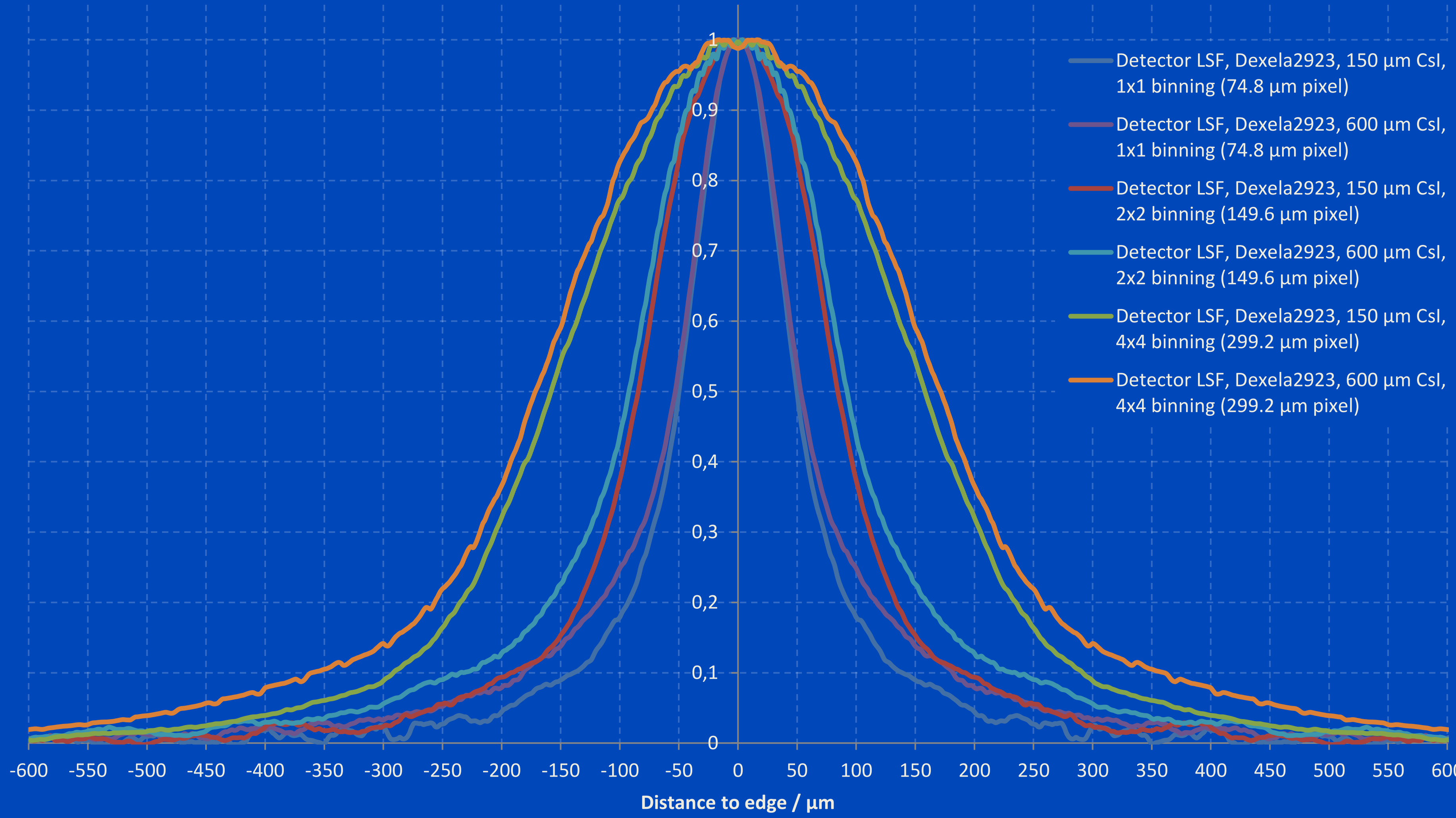
Measurement of the LSF

- Measurement of contact projection images of a PTW normal
- Calculation of the profile along the edge of the normal
- Calculation of the LSF as the derivative of the profile.



Results

LSF normalized to maximum



Conclusion on Dexela 2923

- As expected, the LSF is slightly broader in case of the 600 µm CsI scintillator.
- The full width at half maximum of the LSFs is summarized in the following table.

Binning	Pixel size	150 µm CsI	600 µm CsI
1 × 1	74.8 µm	100 µm	106 µm
2 × 2	149.6 µm	168 µm	184 µm
4 × 4	299.2 µm	316 µm	342 µm

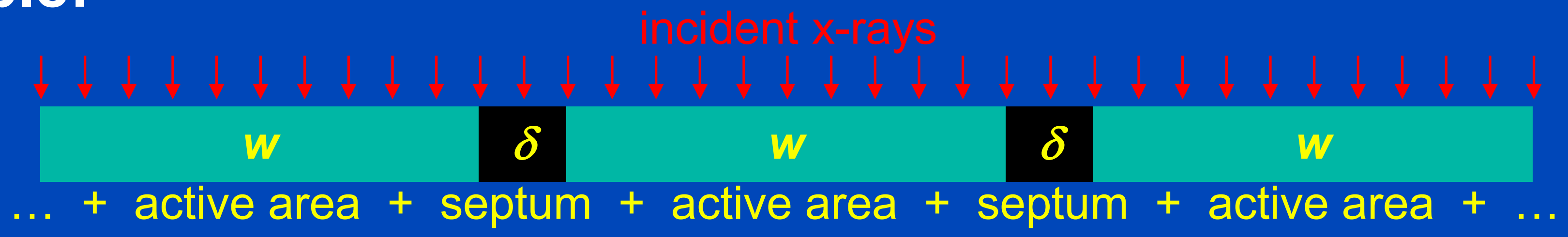
TO BIN OR NOT TO BIN?

System Model

- Object *f*(*x*)
- Presampling function *s*(*x*), normalized to unit area
- Algorithm *a*(*x*), normalized to unit area
- Image *g*(*x*) with

$$g(x) = f(x) * s(x) * a(x) = f(x) * \mathrm{PSF}(x)$$

- Example:



$$s(x) = \mathrm{II}_d^*(x)$$

w = detector pixel width
δ = dead space between pixels

Kachelrieß, Kalender. Med. Phys. 32(5):1321-1334, May 2005

To Bin or not to Bin?
(the continuous view)

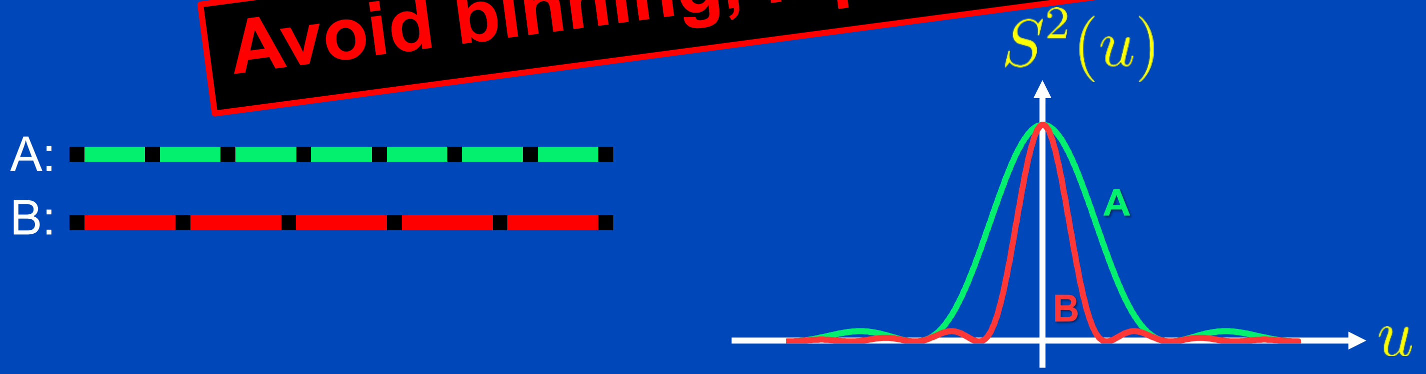
This nice phrase was coined by Norbert Pelc.

- **We have** $\mathrm{PSF}(x) = s(x) * a(x)$ **and** $\mathrm{MTF}(u) = S(u)A(u)$.
- **From Rayleigh‘s theorem we find noise is**

$$\sigma^2 = \int dx\, a^2(x) = \int du\, A^2(u) = \int du\, \frac{\mathrm{MTF}^2(u)}{S^2(u)}$$

- **Compare small (A) with large (B)**

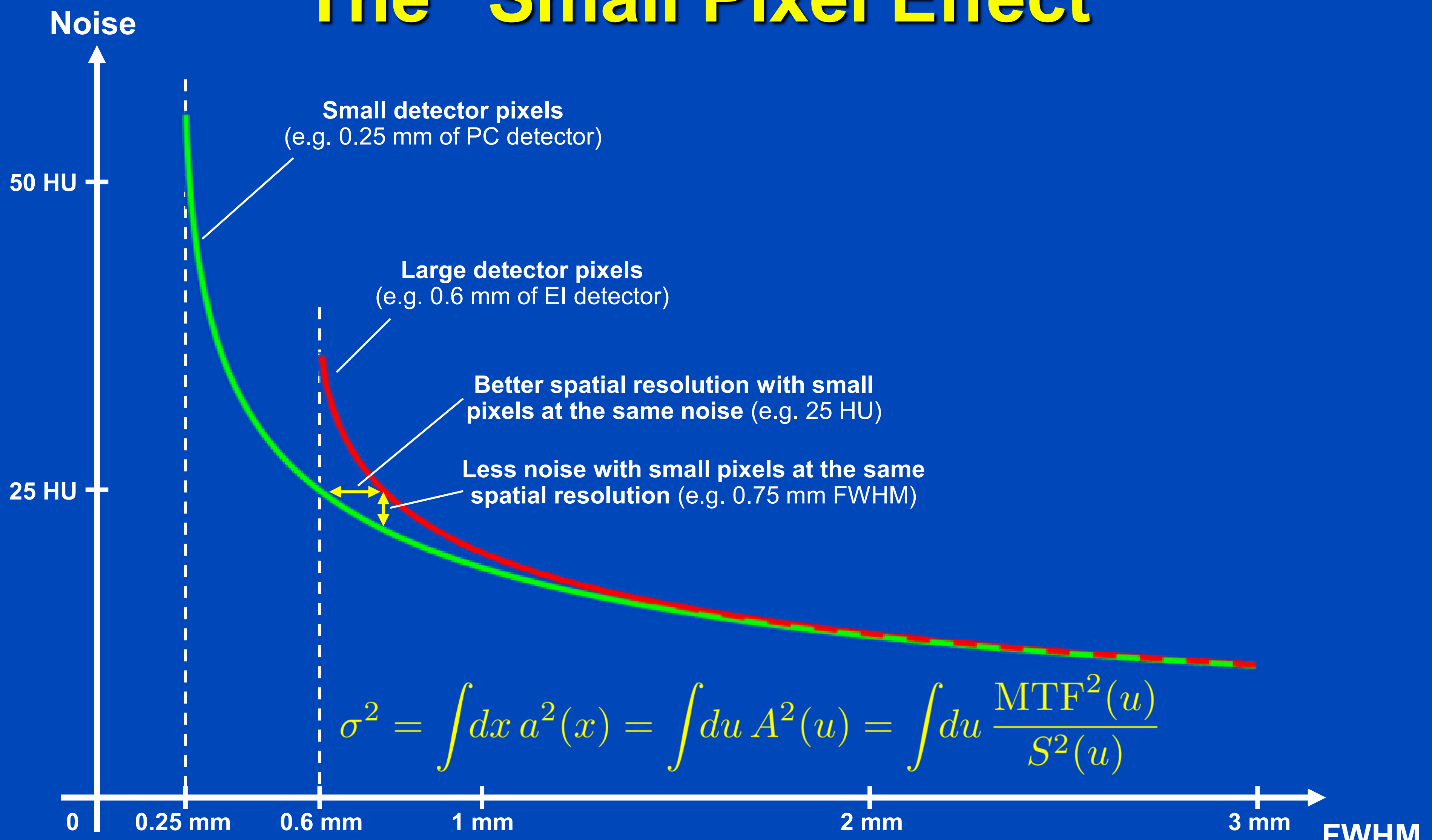
Avoid binning, if possible!



- **We have** $S_\mathrm{A}(u) > S_\mathrm{B}(u)$ **and thus** $\sigma_\mathrm{A}^2 < \sigma_\mathrm{B}^2$.
- **A desired MTF may be best achieved with smaller pixels.**

Kachelrieß, Kalender. Med. Phys. 32(5):1321-1334, May 2005
Baek, Pineda, and Pelc. PMB 58:1433-1446, 2013

The "Small Pixel Effect"



Kachelrieß, Kalender. Med. Phys. 32(5):1321-1334, May 2005

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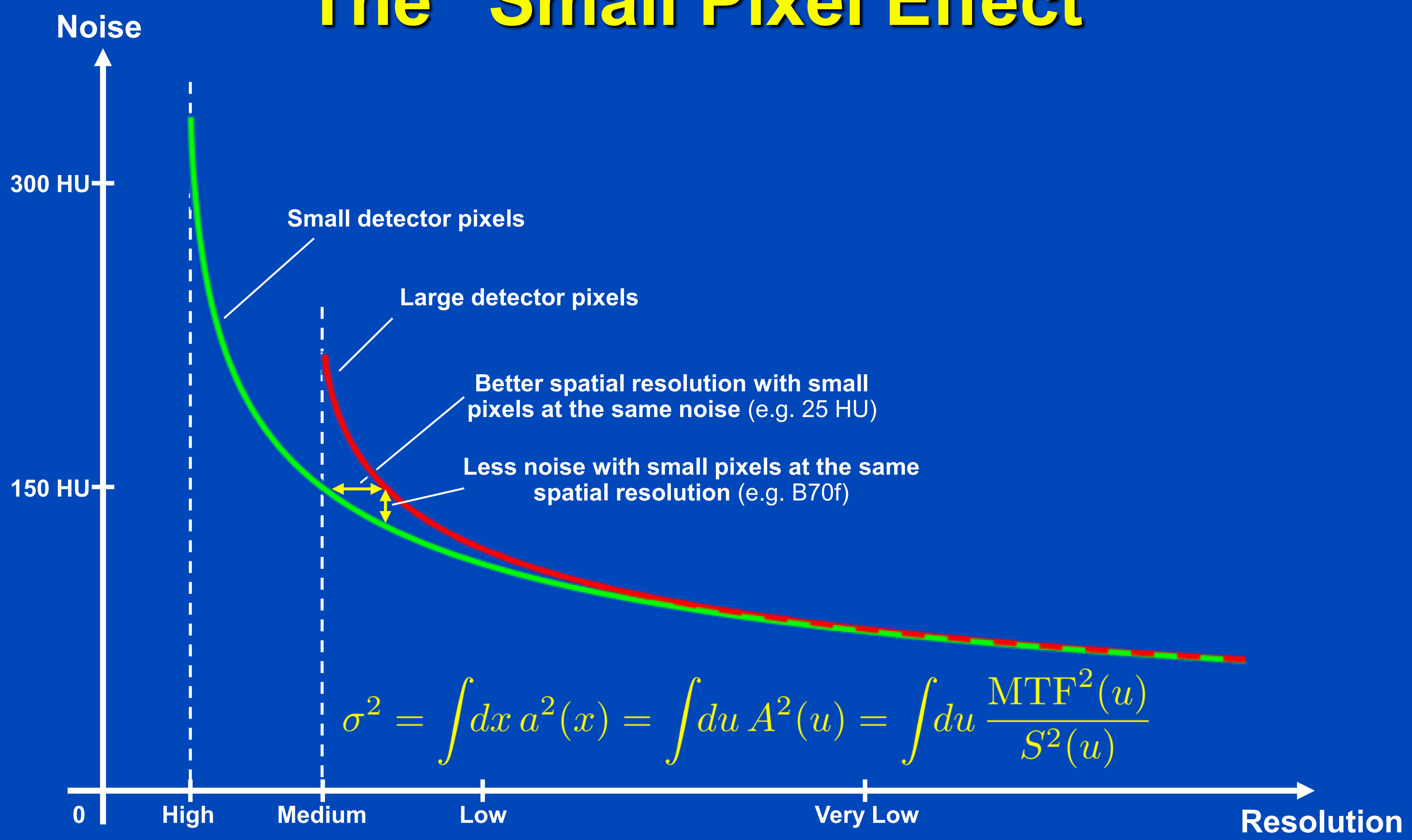
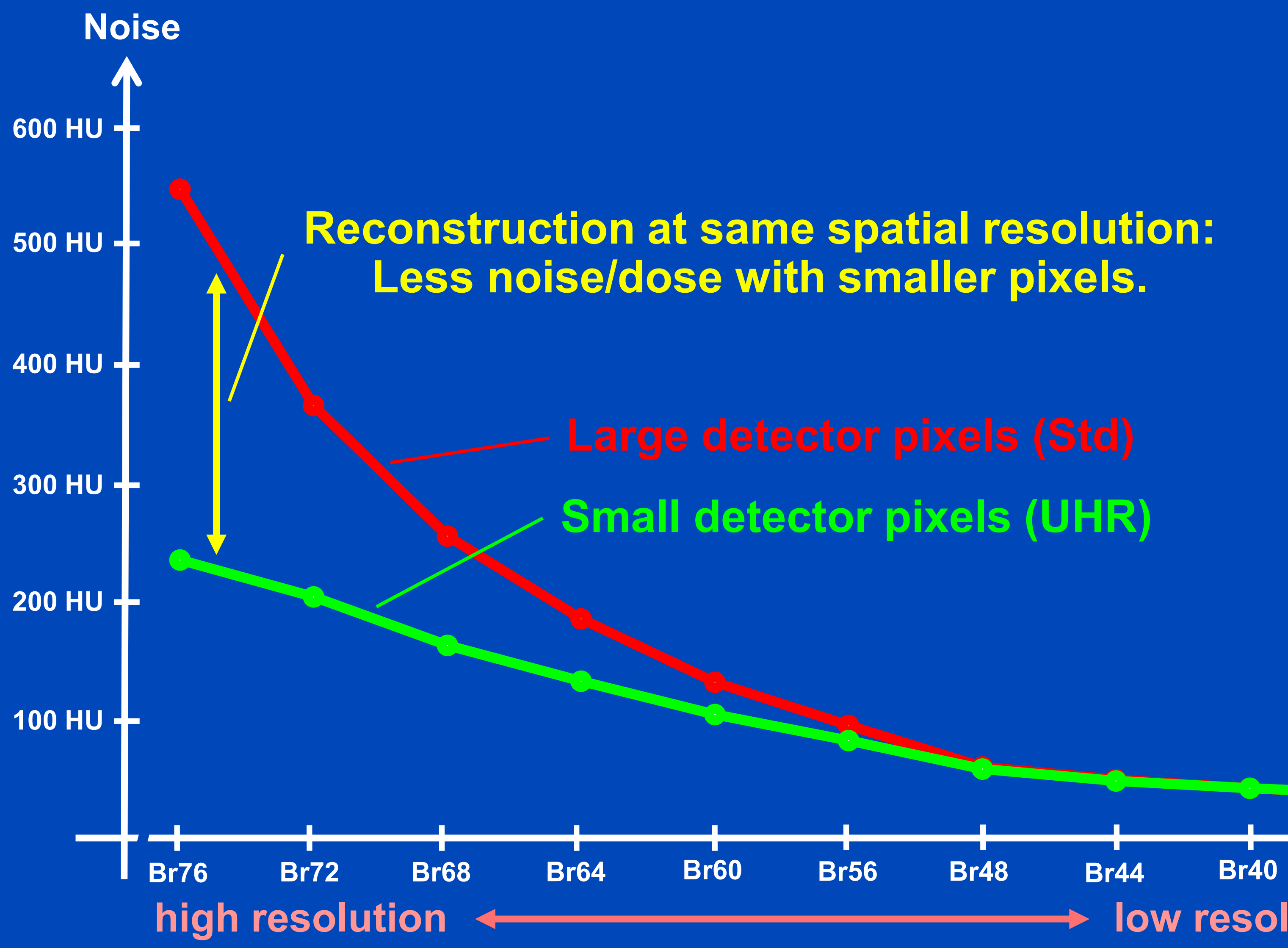
The "Small Pixel Effect"

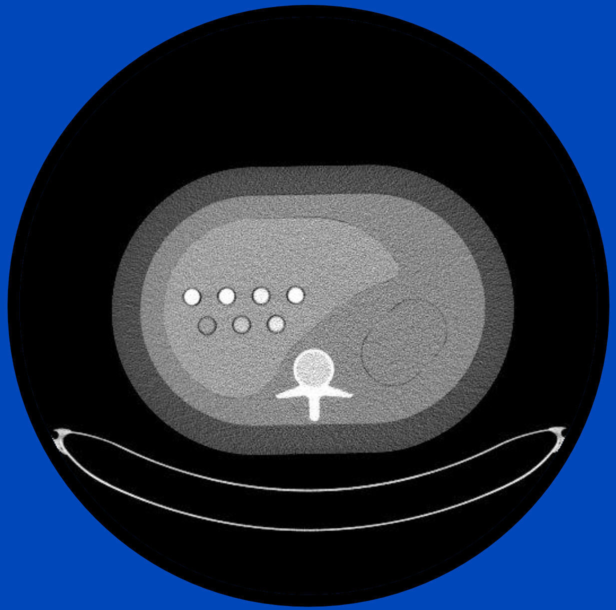


$$\sigma^2 = \int dx\, a^2(x) = \int du\, A^2(u) = \int du\, \frac{\mathrm{MTF}^2(u)}{S^2(u)}$$

Small Pixel Effect at Naeotom Alpha

Medium Phantom, 4 mGy $CTDI_{32}$





To disable the longitudinal small pixel effect, we reconstructed rather thick slices (1 mm thickness).

Longitudinal Small Pixel Effect

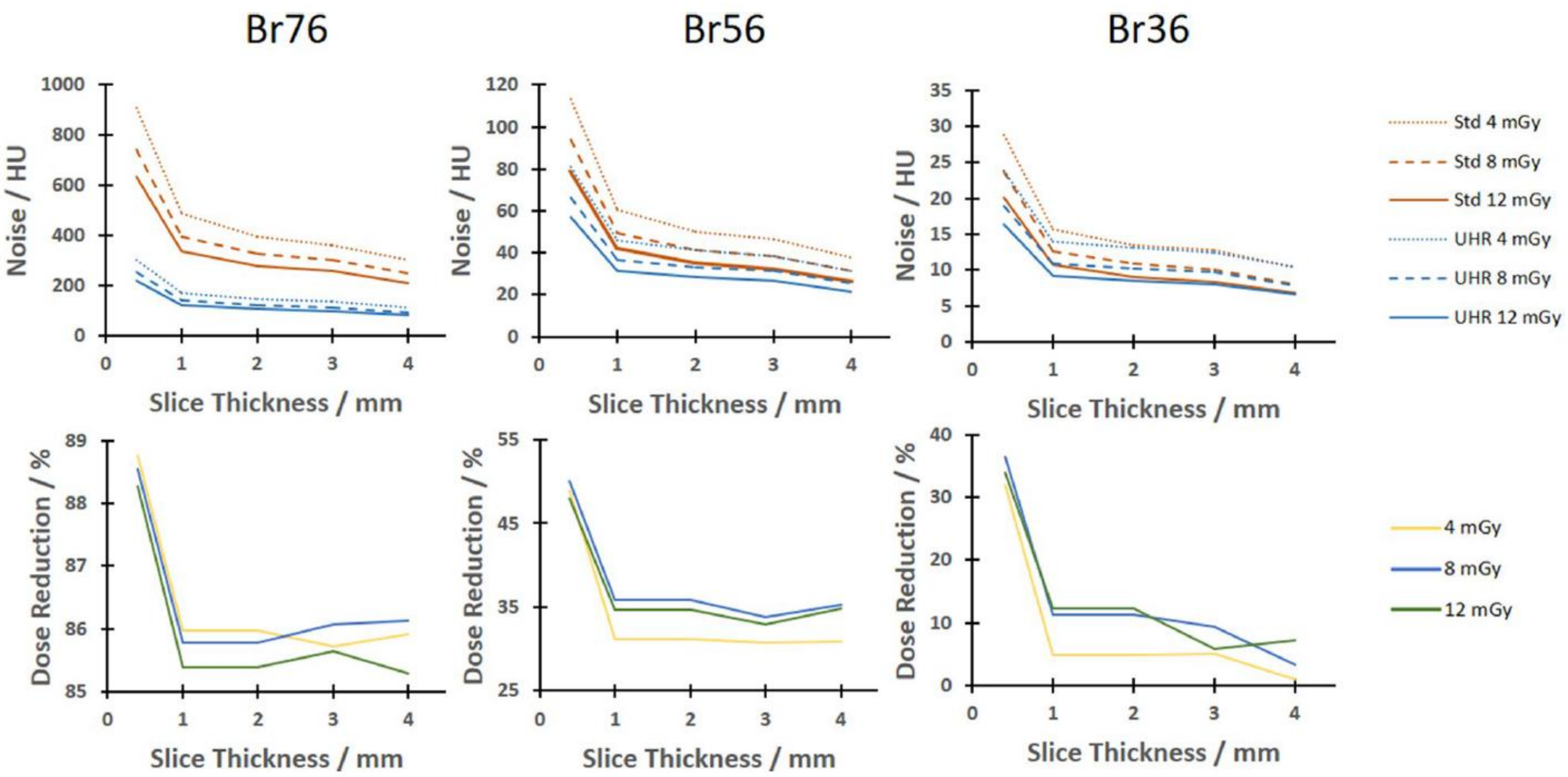


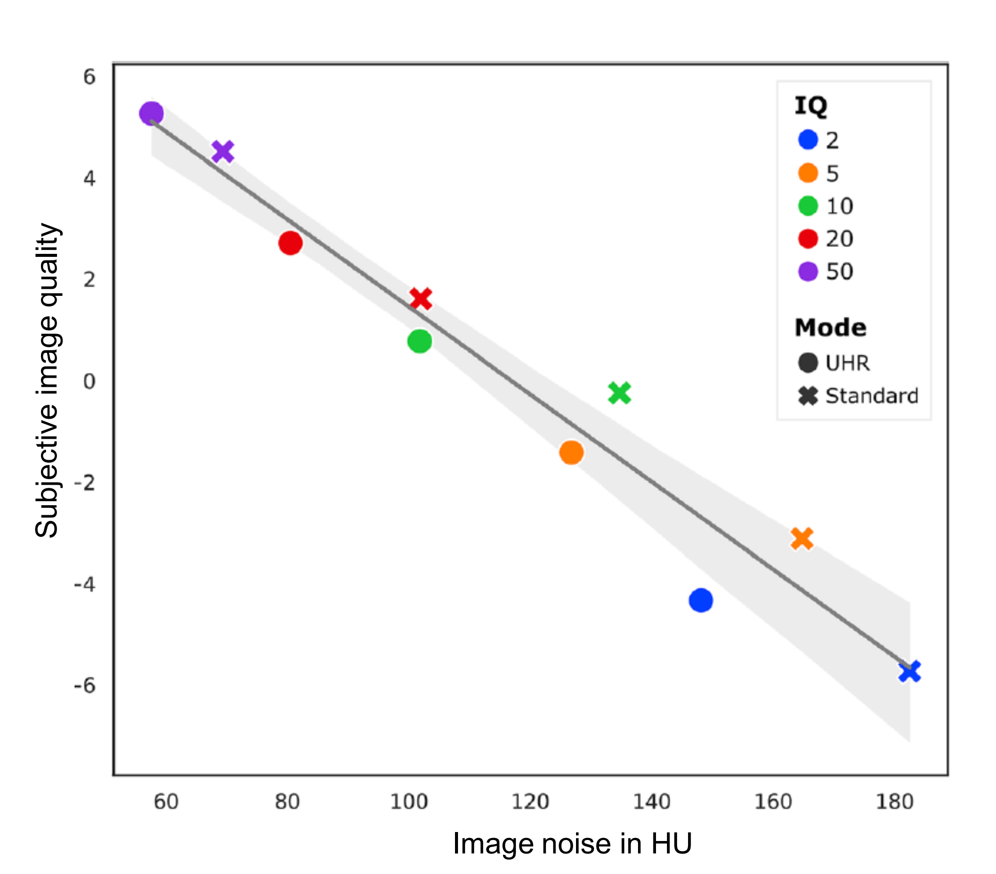
Fig. 3 Image noise as function of slice thickness for the Br76 kernel (top left), the Br56 kernel (top middle), and the Br36 kernel (top right) for all investigated dose levels in the S-phantom. Similarly, the bottom row shows the potential dose reduction for these scenarios

S. Sawall, M. Kachelrieß, et al. Potential radiation dose reduction in clinical photon-counting CT by the small pixel effect: ultra-high resolution (UHR) acquisitions reconstructed to standard resolution. Eur. Rad. 34:4484-4491, 2024.

Small Pixel Effect in PCCT Lung Scans[1]

- UHR vs. Std scan protocols
- 100 kV Sn
- Dose-matched
- Several IQ levels
- Cadaveric specimen
- **About 40% less dose in UHR** (corresponding to ≈23% less noise)

UHR Mode	Image Noise, HU	Standard Mode	Image Noise, HU	Small Pixel Effect
UHR IQ2	148.11 ± 12.73	IQ2	182.46 ± 10.40	−18.83%
UHR IQ5	126.76 ± 16.66	IQ5	164.74 ± 13.92	−23.05%
UHR IQ10	101.77 ± 11.81	IQ10	134.69 ± 17.41	−24.44%
UHR IQ20	80.47 ± 12.81	IQ20	101.93 ± 17.40	−21.05%
UHR IQ50	57.60 ± 8.56	IQ50	79.33 ± 12.64	−27.39%



[1]Huflage et al. Investigating the small pixel effect in ultra-high resolution photon-counting CT of the lung. Invest Radiol 59(4):293-297, April 2024.

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To Bin or not to Bin?[1]

(the discrete view, LI)

- Let detector B be the 2-binned version of detector A:

$$B_{2n} = \tfrac{1}{2}(A_{2n} + A_{2n+1}) \qquad \operatorname{Var} B = \tfrac{1}{2}\operatorname{Var} A$$

- Assume LI to be used to find in-between pixel values. Wlog we may then consider B to be upsampled with mid-point inter… …lation to the pixel size of detector A: …

$$\hat{B} \ldots \ldots)$$

- To obtain t… …r we need to convolve A…

$$a = \tfrac{1}{2}(1,\ 1) * \tfrac{1}{4}(1,\ 2,\ 1) = \tfrac{1}{8}(1,\ 3,\ 3,\ 1)$$

- Noise propagation yields 20% more noise (variance) for the binned detector[2]:

$$\operatorname{Var}\hat{A} = \tfrac{20}{64}\operatorname{Var}A = \tfrac{5}{16}\operatorname{Var}A$$

$$\operatorname{Var}\hat{B} = \tfrac{3}{8}\ \operatorname{Var}A = \tfrac{6}{5}\operatorname{Var}\hat{A} = 1.2\operatorname{Var}\hat{A}$$

20% more noise variance may be compensated by 20% more x-ray dose. Any alternative? Yes:

Avoid binning, if possible!

In 2D binning implies 44% more noise variance or dose. Again, the answer is: „not to bin“.

[1] Kachelrieß, Kalender. Med. Phys. 32(5):1321-1334, May 2005
[2] Noise consideration valid for uncorrelated pixel noise in the unbinned detector.

To Bin or not to Bin?[1]

(the discrete view, NN)

- Let detector B be the 2-binned version of detector A:

$$B_{2n} = \tfrac{1}{2}(A_{2n} + A_{2n+1}) \qquad \mathrm{Var}\,B = \tfrac{1}{2}\mathrm{Var}\,A$$

- Let us now do an upsampling of the detector B such that each of B´s pixels becomes two pixels with the same value and with the pixel size of detector A:

- To obtain t… we need to convolve A…

$$\boldsymbol{a} = \tfrac{1}{4}\left(1,\ 2,\ 1\right)$$

- Noise propagation yields 30% more noise (variance) for the binned detector[2]:

$$\mathrm{Var}\,\hat{A} = \frac{6}{16}\mathrm{Var}\,A = \frac{3}{8}\mathrm{Var}\,A$$

$$\mathrm{Var}\,\hat{B} = \frac{1}{2}\mathrm{Var}\,A = \frac{4}{3}\mathrm{Var}\,\hat{A} = 1.3\,\mathrm{Var}\,\hat{A}$$

30% more noise variance may be compensated by 30% more x-ray dose. Any alternative? Yes:

Avoid binning, if possible!

In 2D binning implies 69% more noise variance or dose.
Again, the answer is: „not to bin“.

[1] Kachelrieß, Kalender. Med. Phys. 32(5):1321-1334, May 2005
[2] Noise consideration valid for uncorrelated pixel noise in the unbinned detector.

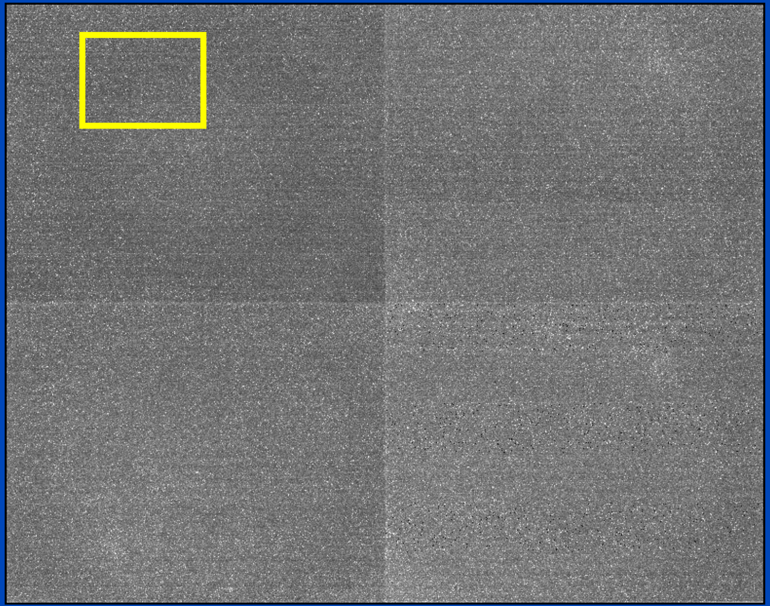
SIMPLE DARK NOISE ASSESSMENT

Aim

- Measurement of the noise of dark images of the PE Dexela 2923 as a function of the integration time.

Measurement of Noise using an ROI?

- Single dark image with ROI $\rightarrow$ Mean, Var
- Assuming ergodicity, the ROI‘s noise would equal the time series‘ noise of a single pixel.
- However, the ROI may also pick up structural fixed pattern “noise”, i.e. structure hidden behind the actual noise, e.g. due to prefilter inhomogeneities or detector sensitivity variations.
- Neglecting the energy-dependence we have



$$\underbrace{S(u,v)}_{\text{signal}} = \underbrace{E(u,v)}_{\text{electronic}} + \underbrace{I(u,v)}_{\text{quantum}}\underbrace{f(u,v)}_{\text{structure}}$$

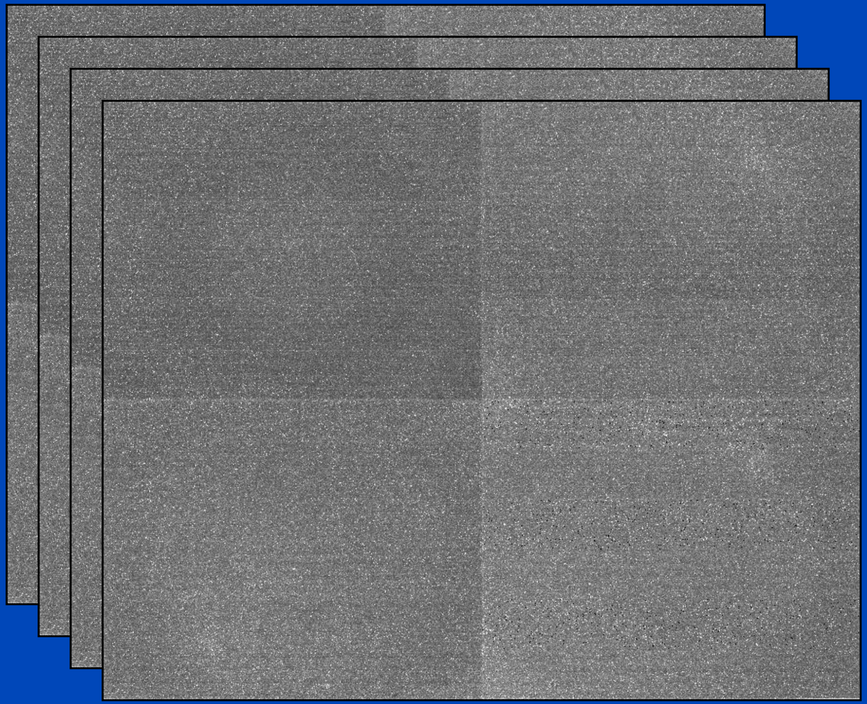
$$\text{e.g. } f(u,v) = e^{-t_{\text{pre}}(u,v)\mu_{\text{pre}}}$$

- From ROI analysis we obtain

$$\mathrm{Var}S = \mathrm{Var}E + (\mathrm{E}f)^2\mathrm{Var}I + (\mathrm{E}I)^2\mathrm{Var}f$$

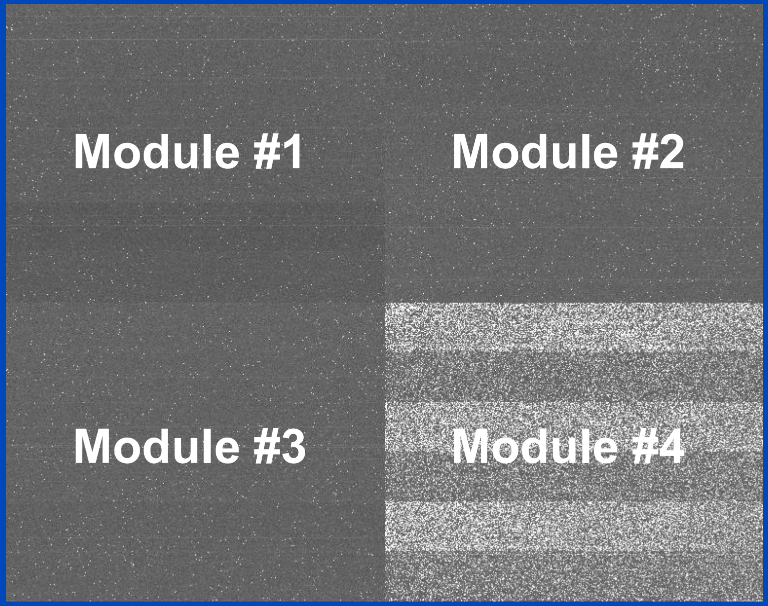
Measurement of Noise

- Measurement of 100 dark images (no binning, 75 µm pixel size).
- Determination of the pixel-wise standard deviation of these images.
- Averaging the standard deviation of all pixels of a module.



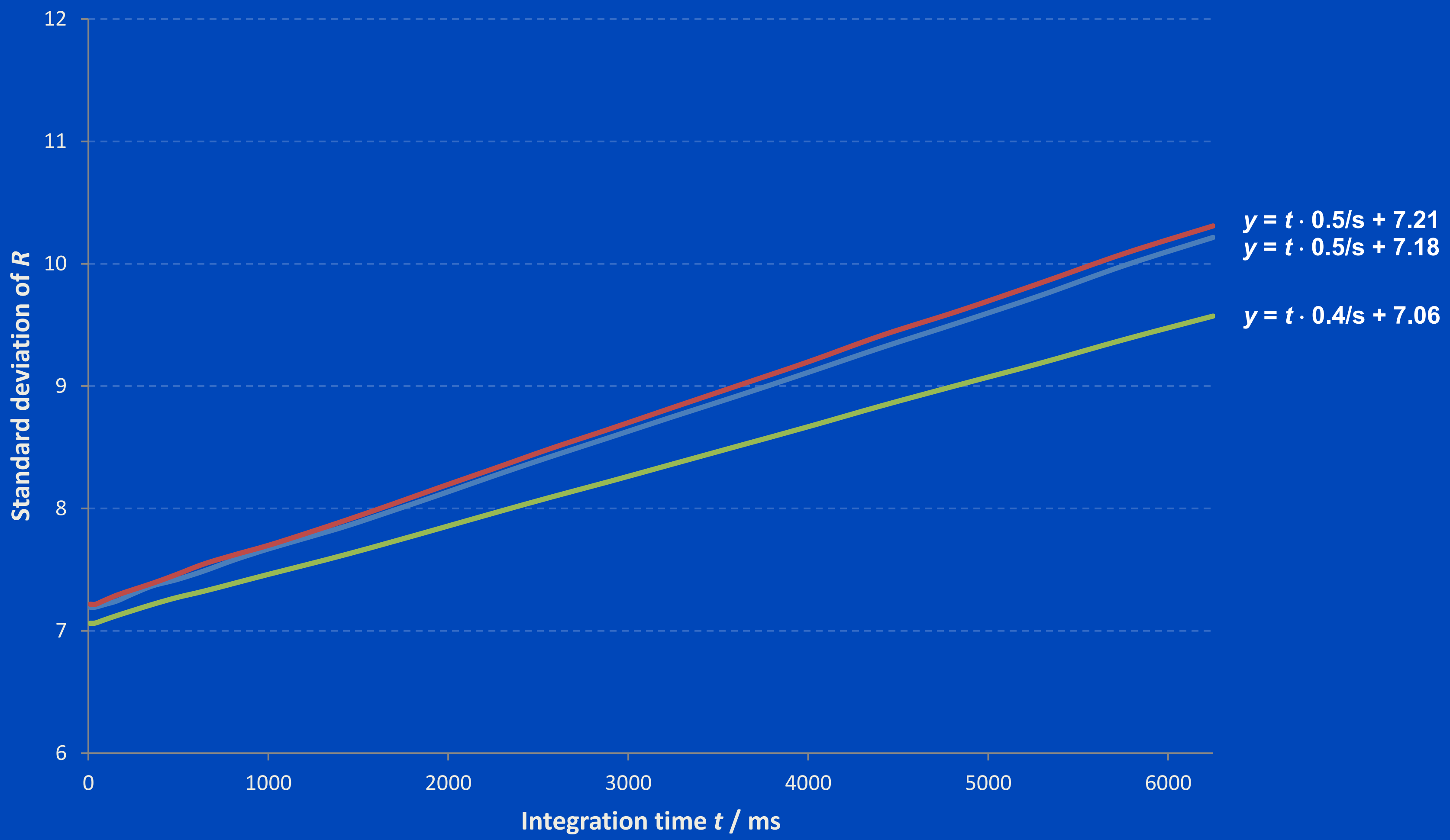
100 dark images

StdDev →



1 sigma image

Electronic Noise *R* as a Function of Integration Time



Plots and linear fits for three different modules of a PerkinElmer flat detector Dexela 2923

Multiple Exposures vs. Long Integration Times

- Assume a signal *S* can be detected only with an added readout noise *R* of known expectation and variance

$$\mathrm{E}(S), \mathrm{Var}(S), \mathrm{E}(R) = 0, \mathrm{Var}(R)$$

- Averaging *T* readouts yields:

$$\mathrm{Var}\Big(\frac{1}{T}\sum_t (S_t + R_t)\Big) = \frac{1}{T^2}\sum_t \Big(\mathrm{Var}S + \mathrm{Var}R\Big) = \frac{\mathrm{Var}S}{T} + \frac{\mathrm{Var}R}{T}$$

- Using the *T*-fold integration time and dividing by *T* yields: $\mathrm{Var}\hat{R} \approx (a + bT)^2$

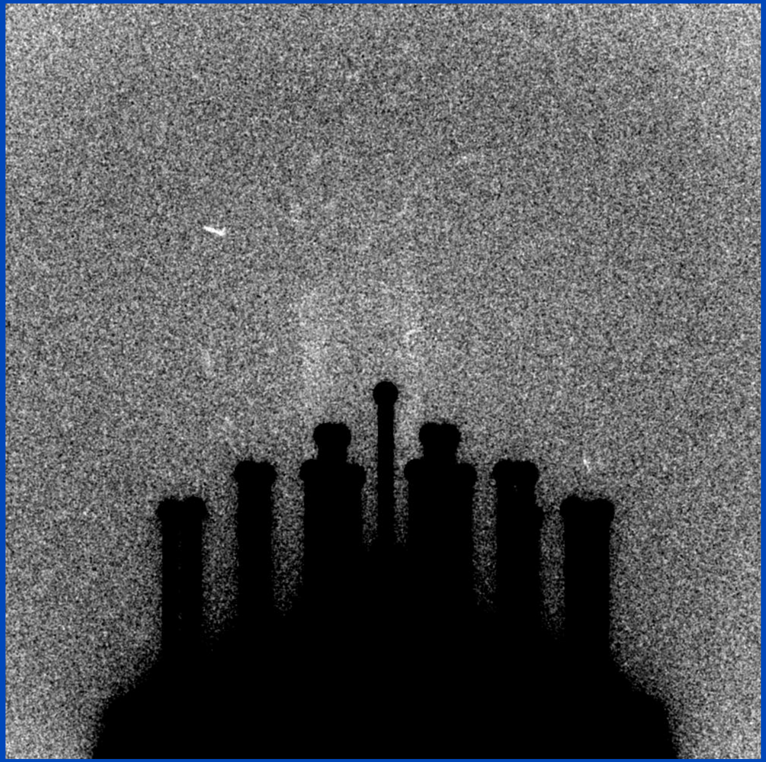
$$\mathrm{Var}\Big(\frac{1}{T}\Big(\sum_t S_t + \hat{R}\Big)\Big) = \frac{1}{T^2}\Big(\sum_t \mathrm{Var}S + \mathrm{Var}\hat{R}\Big) = \frac{\mathrm{Var}S}{T} + \frac{\mathrm{Var}\hat{R}}{T^2}$$

SIMULATION OF REALISTIC NOISE

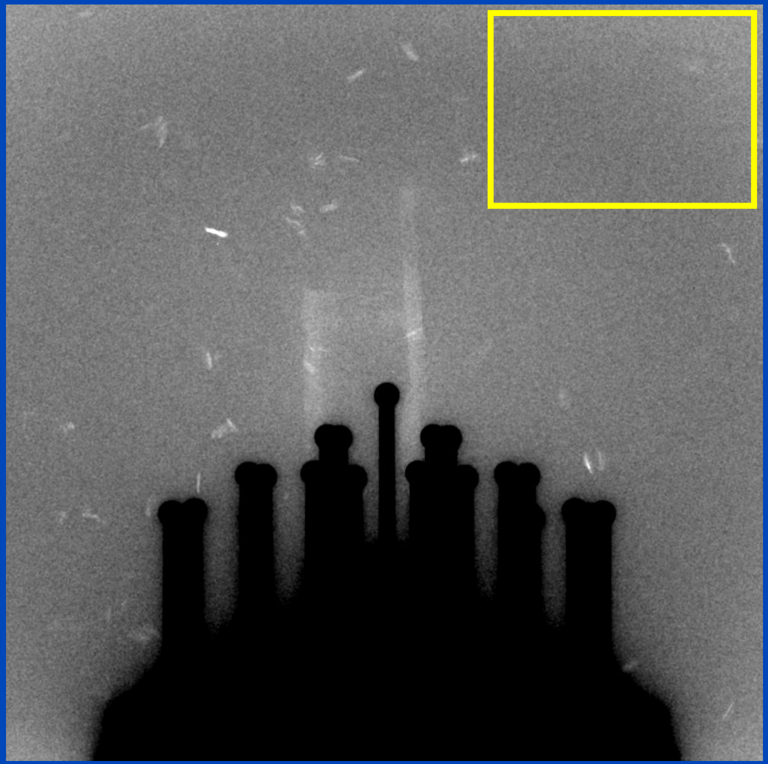
Aim

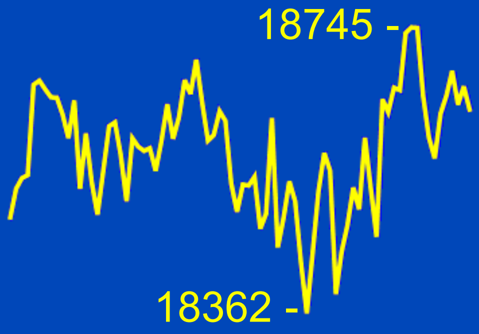
- Simulate realistic projection noise (e.g. to train a denoising CNN)
- Do this given
  - Perkin Elmer XRD 1620 AN3 with CsI scintillator
  - 800 projections under the same projection direction.

First Image (of 80)

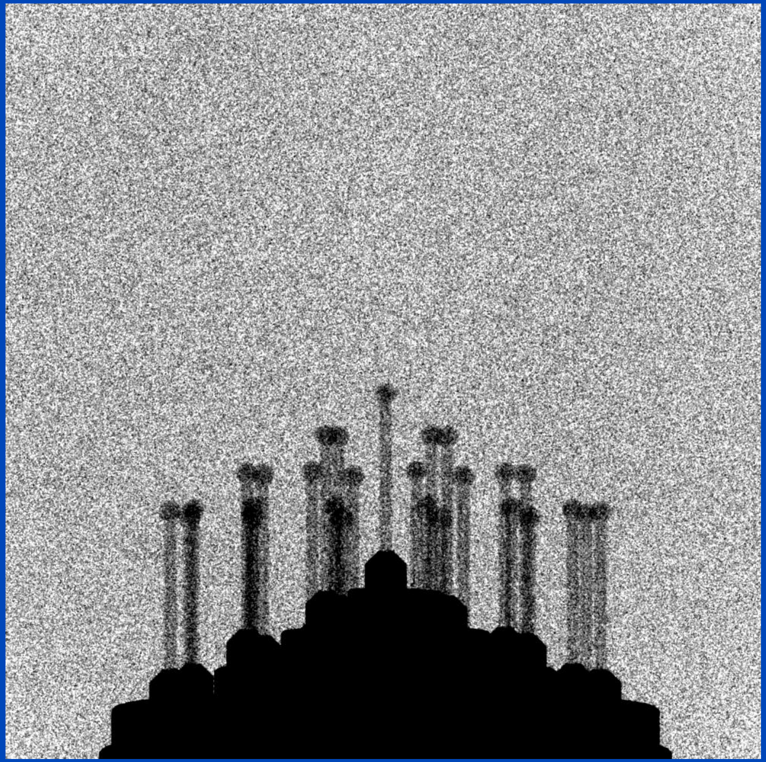


Mean Image (of 80)

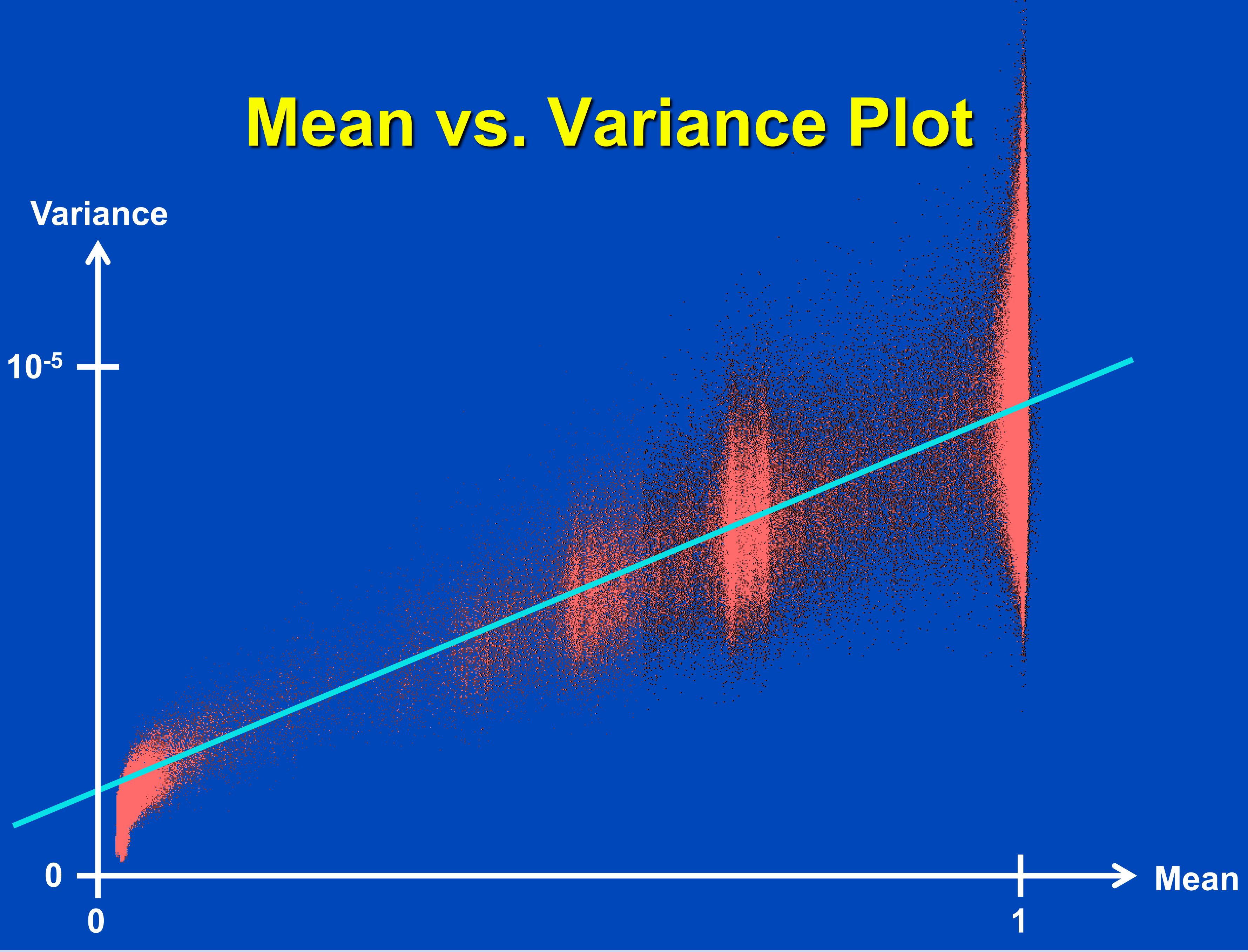




Sigma Image (Std. Dev. of 80 Images)



Mean vs. Variance Plot



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Noise Theory

- **Signal = x-ray intensity times gain plus electronic noise**

$$S = gI + N,\ \ \mathrm{Var}I = \mathrm{E}I,\ \ \mathrm{E}N = 0$$

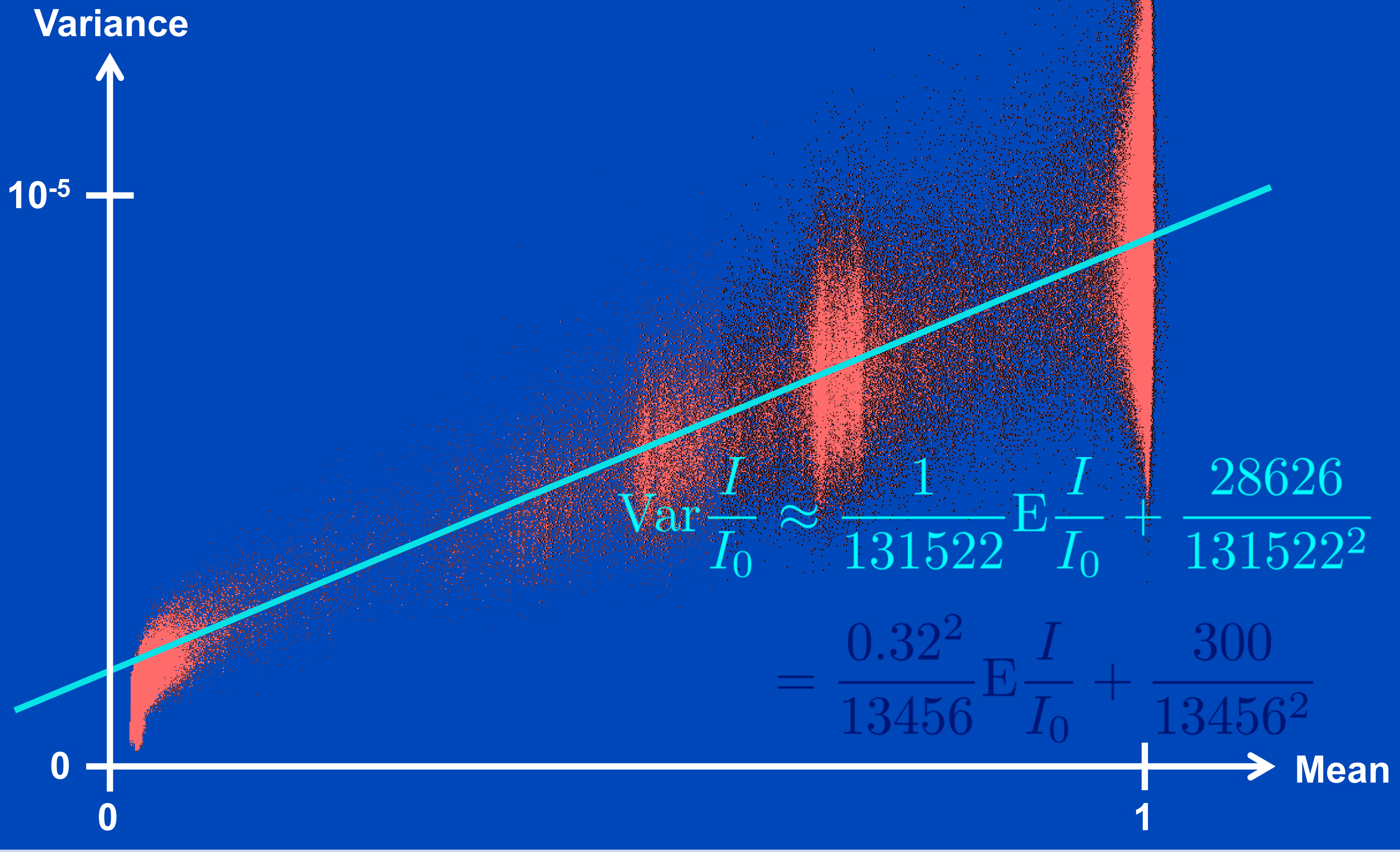
- **Noise in relative signal is**

$$\mathrm{Var}\frac{S}{S_0} = \frac{\mathrm{Var}S}{S_0^2} = \frac{\mathrm{Var}S}{g^2 I_0^2} = \frac{\mathrm{Var}I}{I_0^2} + \frac{\mathrm{Var}N}{g^2 I_0^2} =$$
$$= \frac{1}{I_0}\mathrm{E}\frac{I}{I_0} + \frac{const.}{I_0^2} \approx \frac{1}{131522}\mathrm{E}\frac{I}{I_0} + \frac{28626}{131522^2}$$

- **Noise in projection values is**

$$\mathrm{Var}\,p = \mathrm{Var}\ln\frac{S}{S_0} = \frac{1}{(\mathrm{E}I/I_0)^2}\mathrm{Var}\frac{S}{S_0} = \frac{1}{I_0}e^{p} + \frac{const.}{I_0^2}e^{2p}$$

Mean vs. Variance Plot

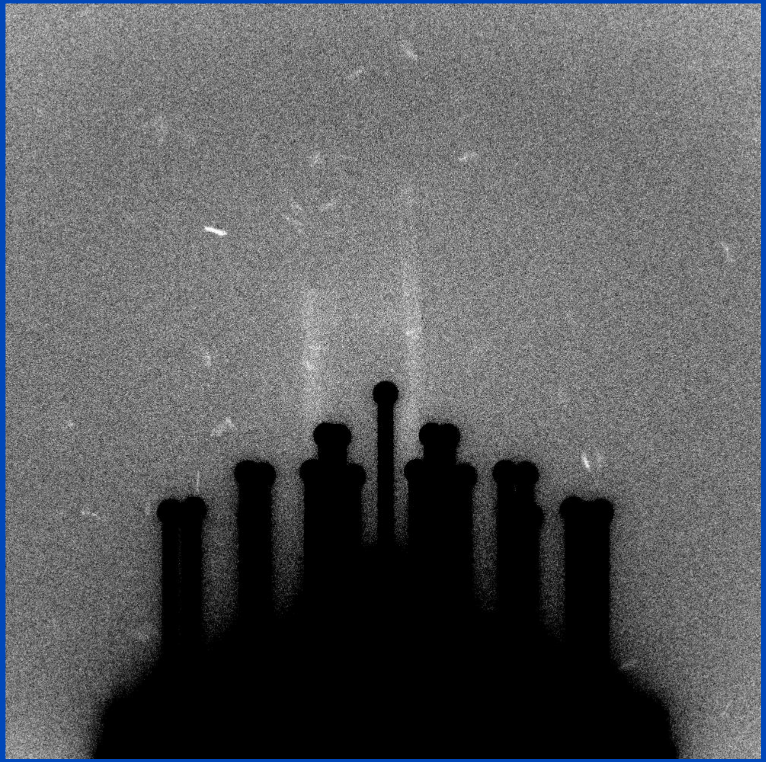


$$\mathrm{Var}\frac{I}{I_0} \approx \frac{1}{131522}\mathrm{E}\frac{I}{I_0} + \frac{28626}{131522^2}$$

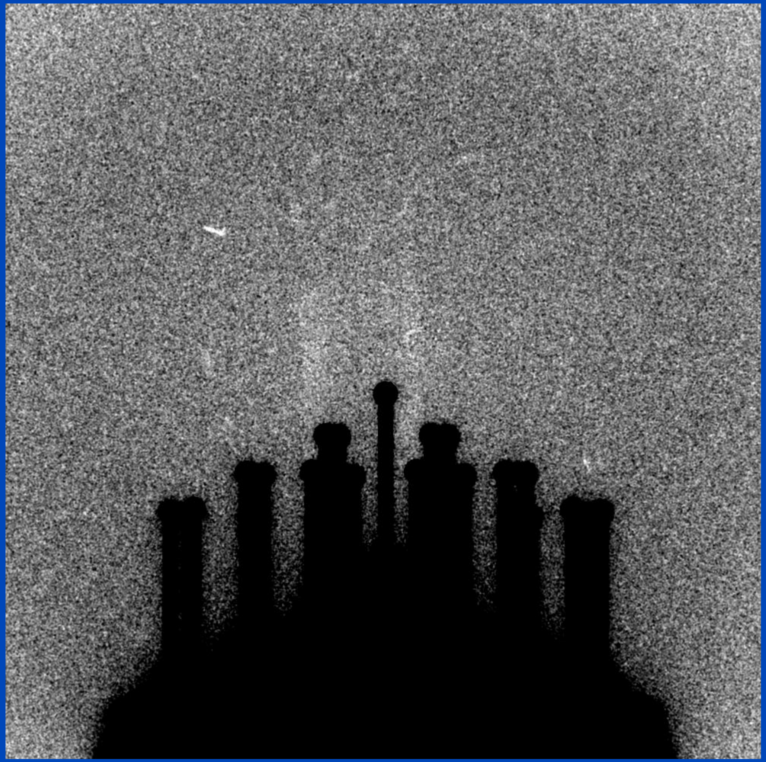
$$= \frac{0.32^2}{13456}\mathrm{E}\frac{I}{I_0} + \frac{300}{13456^2}$$

Mean Image (of 80)

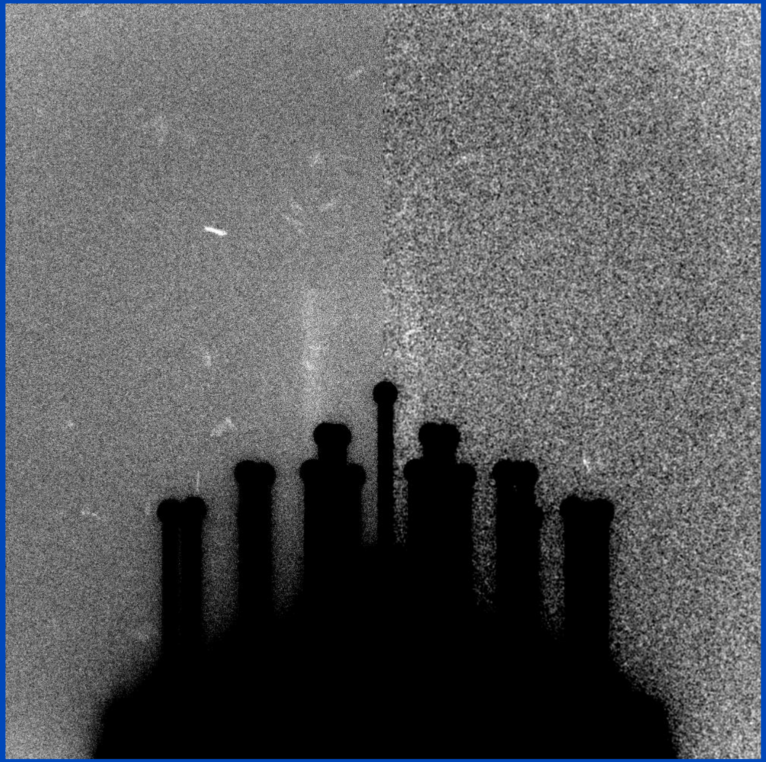
Mean + Noise



First Image (of 80)



Mean + Noise vs. First Image



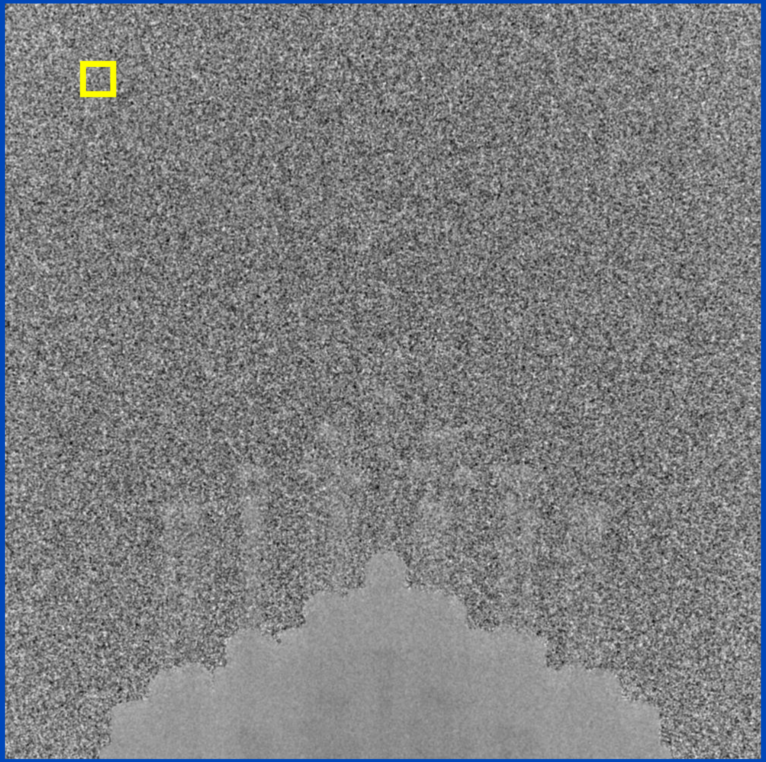
Well-Known NPS Theory

$$\mathcal{L}f(x) = \mathcal{L}f(y),\ \mathrm{E}f(x) = 0,\ \mathrm{E}f(x)f(y) = \sigma^2\delta(x-y)$$

$$g(x) = f(x) * k(x),\ k(x) = k(-x)$$

$$\begin{aligned}\mathrm{E}g(x)g(y) &= \mathrm{E}\int du\, dv\, f(u)f(v)k(x-u)k(y-v)\\ &= \sigma^2\int du\, k(x-u)k(y-u)\\ &= \sigma^2\int dw\, k(w)k(y-x+w)\\ &= \sigma^2 k(w) * k(w)\big|_{w=x-y}\end{aligned}$$

Noise of First Image



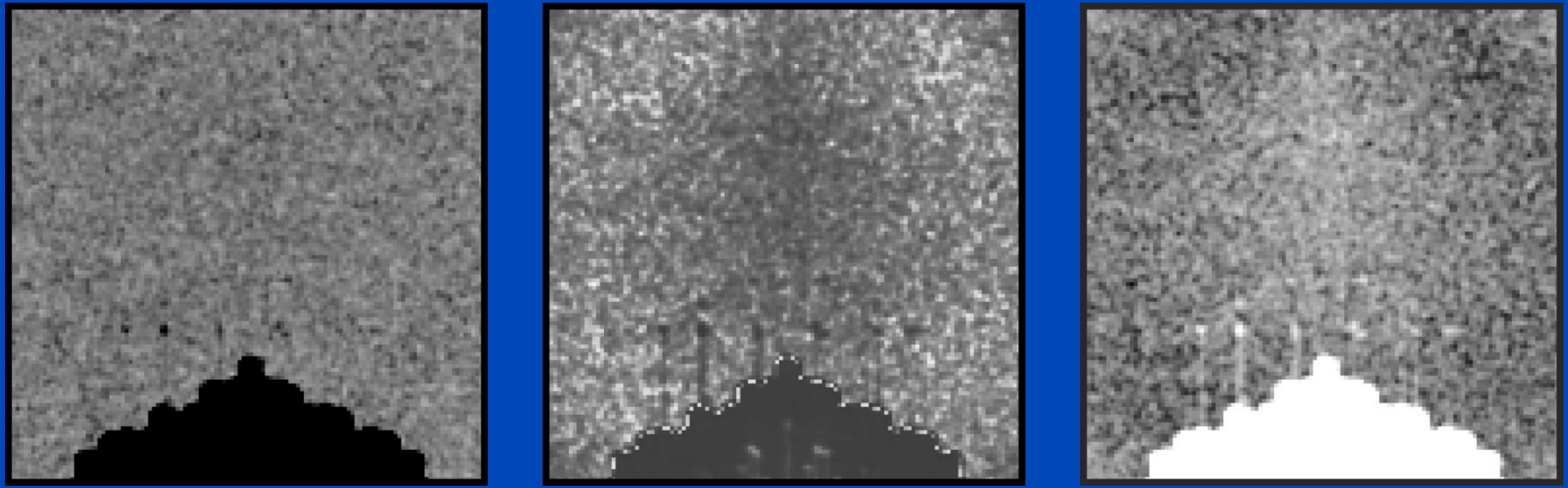
$$g(x) = f(x) * k(x),\ \mathrm{E}f(x) = 0,\ \mathrm{E}g(x) = 0$$

PSF Assessment

- Take a 15×15 patch and compute its autocorrelation.
- Do this in all 80 realizations and average the results.
- This is the autocorrelation of the 2D PSF.
- Assume separability, average columns and rows.
- Remove bias, scale to unit height, average the averages.
- This is the autocorrelation of the 1D PSF.
- From this, determine the PSF.
  - We use the PSF model $(c\ \ b\ \ a\ \ b\ \ c)^T * (c\ \ b\ \ a\ \ b\ \ c)$.
  - Non-linear optimization matches the model to the autocorrelation.
- Repeat the procedure for all possible patch centers.

-2	-2	-3	-2	-1	0	1	1	0	0	0	0	1	1	2
-1	-2	-2	-2	0	1	1	1	0	0	0	0	1	1	1
0	-1	-1	0	1	2	2	1	0	0	0	0	0	0	0
0	-1	0	0	2	3	4	2	1	0	0	0	0	1	0
0	0	0	0	2	4	6	6	4	2	1	1	2	2	0
0	0	0	0	2	8	20	27	20	8	2	1	2	2	0
-1	0	0	0	3	19	52	72	53	20	3	0	1	1	0
-1	0	1	1	4	26	72	100	73	26	3	0	0	0	0
0	1	2	2	3	19	53	73	53	20	2	0	0	0	0
1	2	2	1	1	7	20	28	21	9	1	-1	0	0	0
1	2	2	0	0	0	4	7	7	4	0	-1	-1	-1	-1
0	1	1	0	-1	0	2	4	5	4	1	-1	-2	-2	-2
0	1	0	-1	-2	-1	1	3	4	3	0	-1	-2	-3	-2
0	1	1	0	-2	-2	0	1	2	1	0	-2	-2	-3	-2
0	1	1	-1	-3	-3	-1	0	0	0	-1	-2	-3	-3	-3

abc-Images



a / 13   b / 7   c / 1

On average the PSF is (c   b   a   b   c) = (1   7   13   7   1) / 29.

Adjusting I_0 due to Correlations

$$g(x) = f(x) * k(x)$$

$$\mathrm{Var}\, g(x) = \mathrm{Var}\, f(x) * k(x) = k^2(x) * \mathrm{Var}\, f(x) = \sigma^2 \int dx\, k^2(x)$$

$$\begin{aligned}\kappa^2 &= \int dx\, dy\, k^2(x,y) \\ &= \int dx\, dy\, k^2(x) k^2(y) \\ &= 0.32^2\end{aligned}$$

After smoothing, image noise will 32% of the desired value. Consequently the number of primary quanta I_0 that are used for simulation must be adjusted.

Noise Theory

- **Signal = x-ray intensity times gain plus electronic noise**

$$S = gI + N,\ \ \mathrm{Var}\,I = \kappa^2\,\mathrm{E}I,\ \ \mathrm{E}N = 0$$

- **Noise in relative signal is**

$$\mathrm{Var}\frac{S}{S_0} = \frac{\mathrm{Var}\,S}{S_0^2} = \frac{\mathrm{Var}\,S}{g^2 I_0^2} = \frac{\mathrm{Var}\,I}{I_0^2} + \frac{\mathrm{Var}\,N}{g^2 I_0^2} =$$

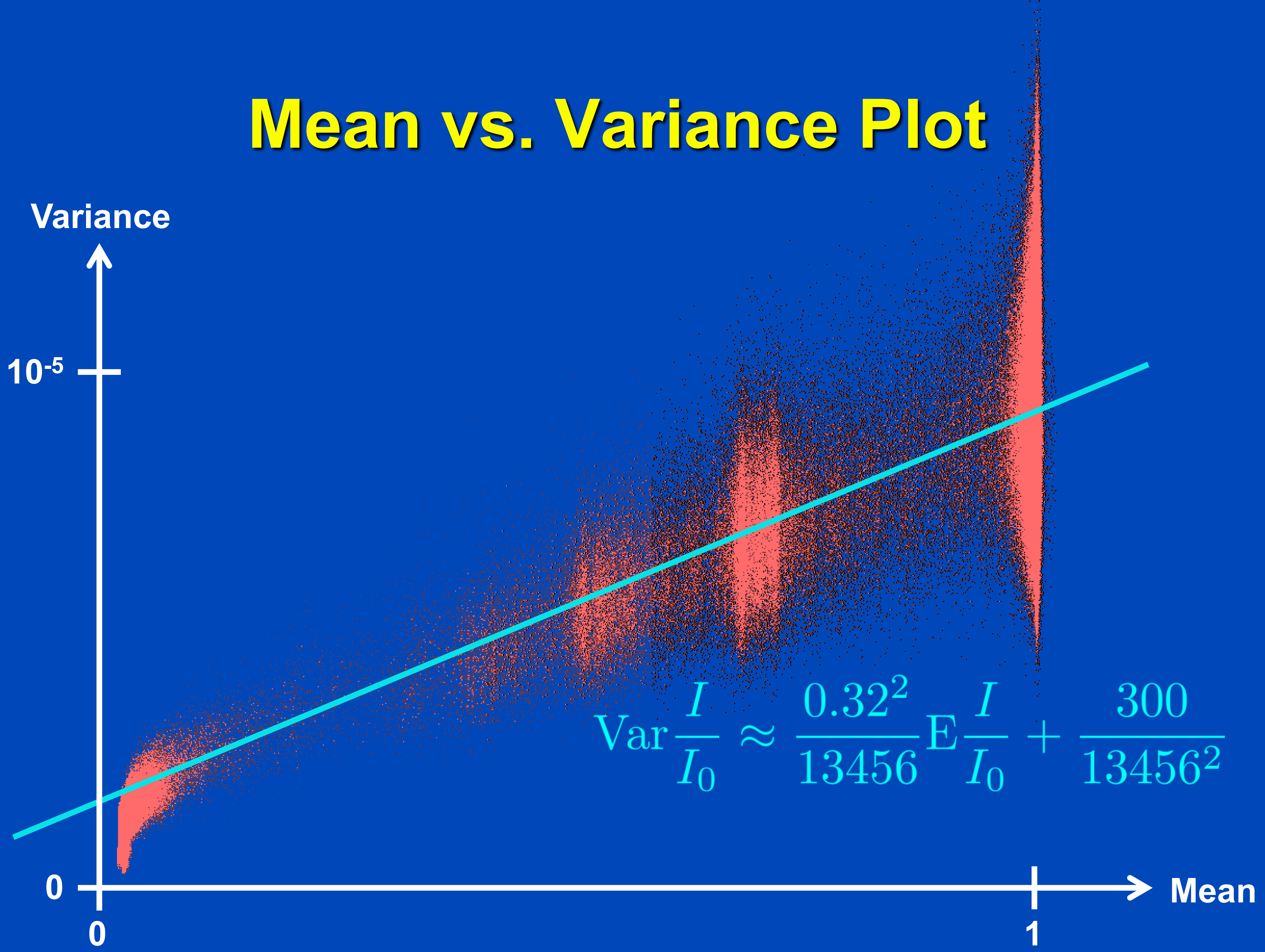
$$= \frac{\kappa^2}{I_0}\mathrm{E}\frac{I}{I_0} + \frac{const.}{I_0^2} \approx \frac{0.32^2}{13456}\mathrm{E}\frac{I}{I_0} + \frac{300}{13456^2}$$

- **Noise in projection values is**

$$\mathrm{Var}\,p = \mathrm{Var}\ln\frac{S}{S_0} = \frac{1}{(\mathrm{E}I/I_0)^2}\mathrm{Var}\frac{S}{S_0} = \frac{\kappa^2}{I_0}e^{p} + \frac{const.}{I_0^2}e^{2p}$$

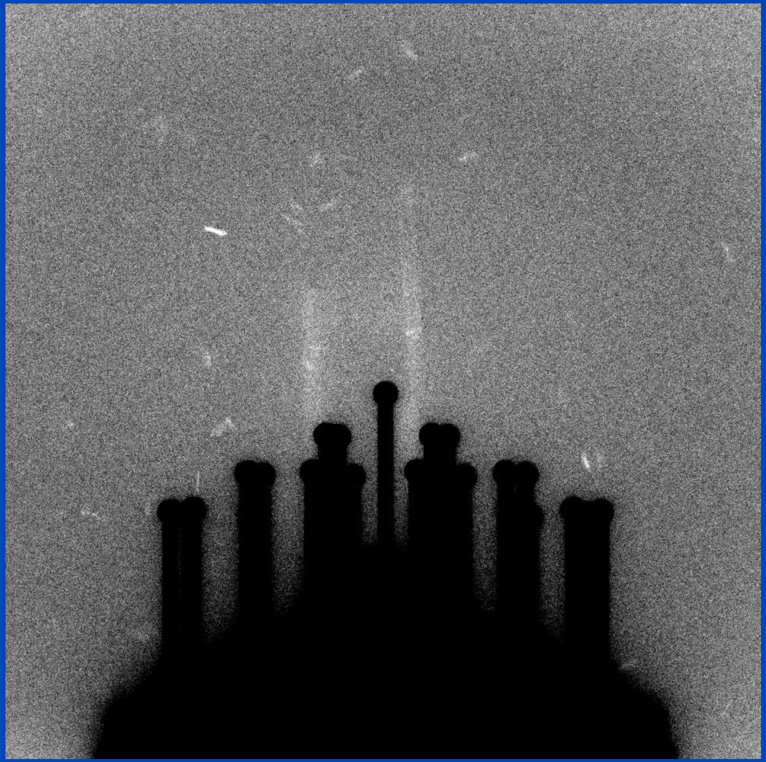
dkfz.

Mean vs. Variance Plot

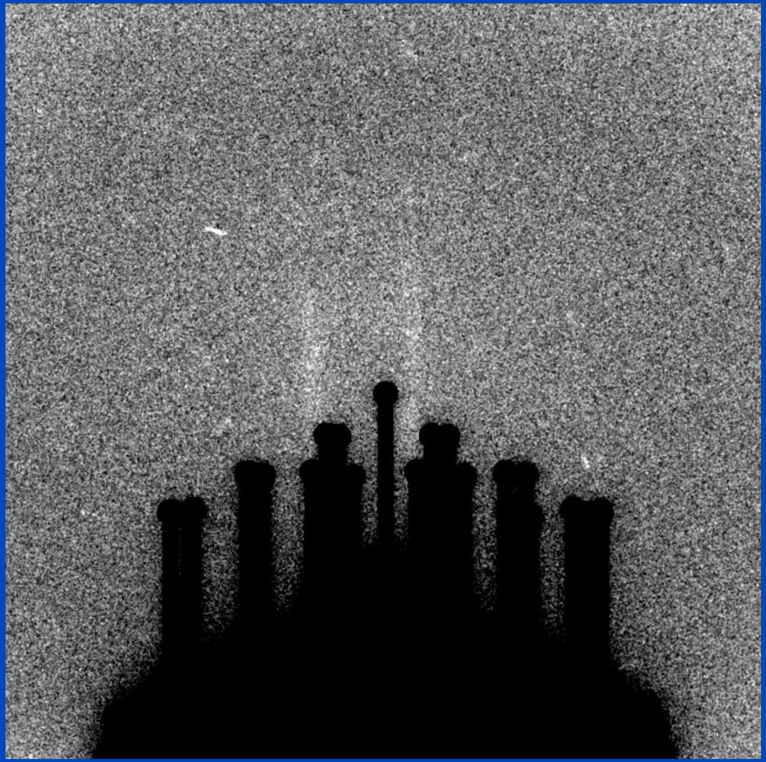


$$\operatorname{Var}\frac{I}{I_0} \approx \frac{0.32^2}{13456}\operatorname{E}\frac{I}{I_0} + \frac{300}{13456^2}$$

Mean + Noise

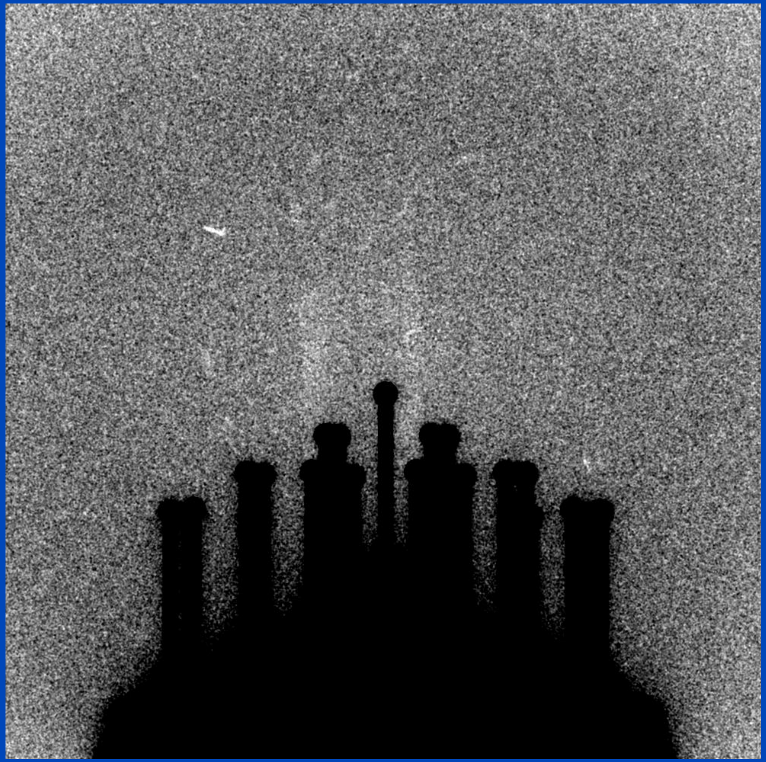


Mean + Correlated Noise

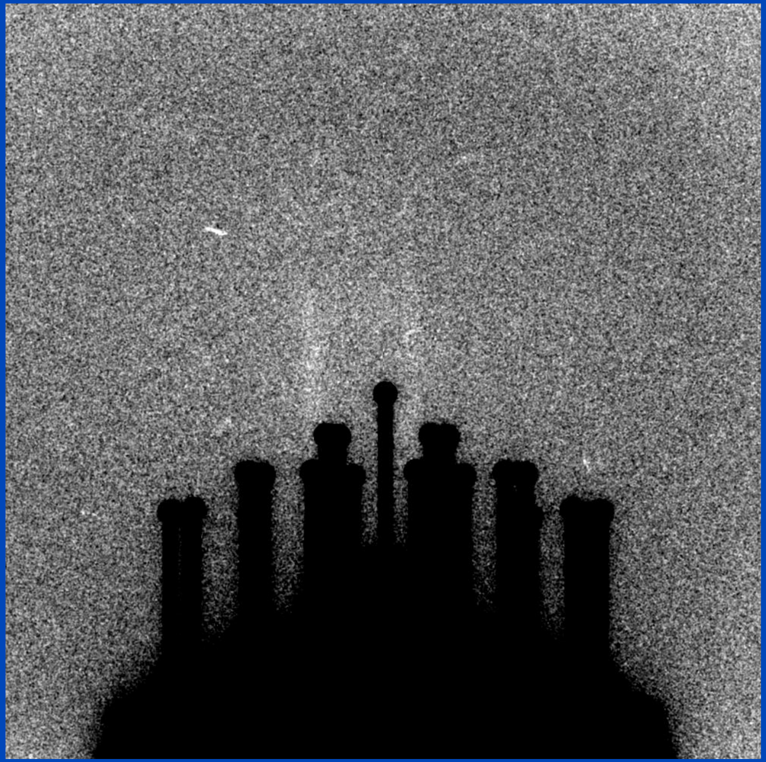


Noise convolved with (1 7 13 7 1) / 29 before added to the mean image.

First Image (of 80)



Mean + Correlated Noise vs. First Image



Limitations

- The data were available only in 2×2 binning mode.
- Dark images were not available.
- Noise distribution was not analyzed (only mean and sigma)

Summary

- The PSF of our detector can be approximated by convolving the rows and columns with the FIR kernel (1   7   13   7   1).
- This appears to represent samples of a triangle function with FWHM = 7/3 = 2.333 detector pixels.
- Noise texture can be well reproduced.

Thank You!

- This presentation will soon be available at www.dkfz.de/ct.
- Job opportunities through DKFZ’s international PhD or Postdoctoral Fellowship programs (marc.kachelriess@dkfz.de).
- Parts of the reconstruction software were provided by RayConStruct® GmbH, Nürnberg, Germany.