

# Reconstructing Invariances of CT Image Denoising Networks using Invertible Neural Networks

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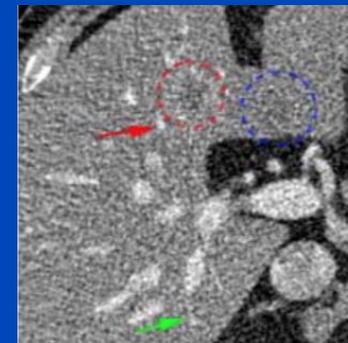
<sup>3</sup>Ludwig Maximilian University of Munich, Germany

# Motivation

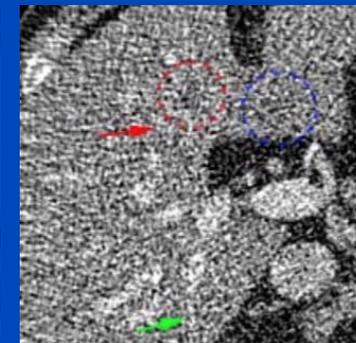
- Deep learning methods are employed for many problems in medical image formation, e.g.
  - Reconstruction
  - Scatter estimation
  - Image-based noise reduction
- Results of DNN-based methods often excel those of conventional algorithms qualitatively and quantitatively
- They lack interpretability due to black-box nature of DNNs → recent advancement in generative modelling signal false confidence

## Here:

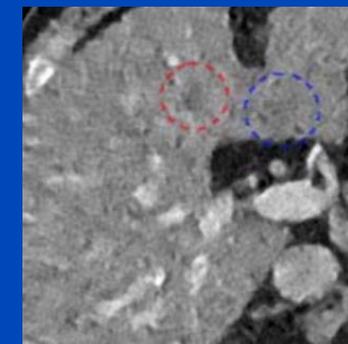
- Not focusing on denoising performance
- Lay fundamentals for post-hoc interpretability and robustness analysis of denoising DNNs
- Investigate what networks learned to represent and to ignore → Their invariances



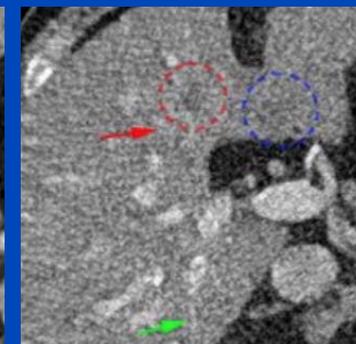
Full-dose reconstruction



Quarter-dose reconstruction



CNN trained with MSE



CNN trained as WGAN with VGG Loss

Examples for Low-dose CT denoising<sup>1</sup>

<sup>1</sup>Q. Yang et al. (2018). Low-Dose CT Image Denoising Using a Generative Adversarial Network With Wasserstein [...]. IEEE TMI.

# Methods

## Deep-learning based CT Denoising

Deep learning-based CT denoising methods aim to find a function  $f(\cdot; \theta)$  (realized by a CNN with parameters  $\theta$ ), s.t.

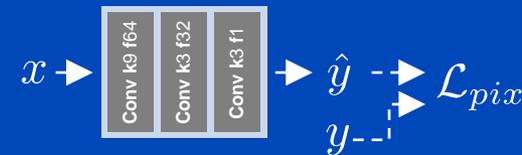
$$\arg \min_{\theta} \|f(x; \theta) - y\|$$

where  $x$  is the low dose input image and  $y$  is the high dose target image.

Recover invariances of two denoising methods:

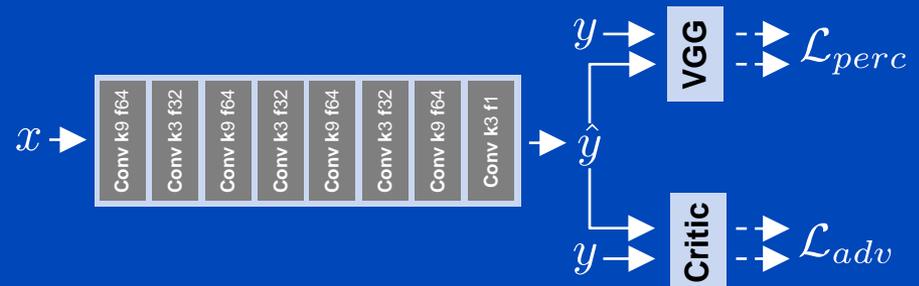
- **Chen et al.<sup>1</sup>:**

- Simple 3-layer CNN
- Trained to with  $\mathcal{L}_2$  loss
- Trained on patches of size  $33 \times 33$  px<sup>2</sup>



- **Yang et al.<sup>2</sup>:**

- 8-layer CNN as generator
- Trained as Wasserstein GAN (WGAN)
- Additional perceptual loss
- Trained on patches of size  $64 \times 64$  px<sup>2</sup>



<sup>1</sup>H. Chen et al., "Low-dose CT denoising with convolutional neural network", ISBI 2017, 2017.

<sup>2</sup>Q. Yang et al., "Low-Dose CT Image Denoising Using a Generative Adversarial Network [...]", in *IEEE TMI*, vol. 37, no. 6, 2018.

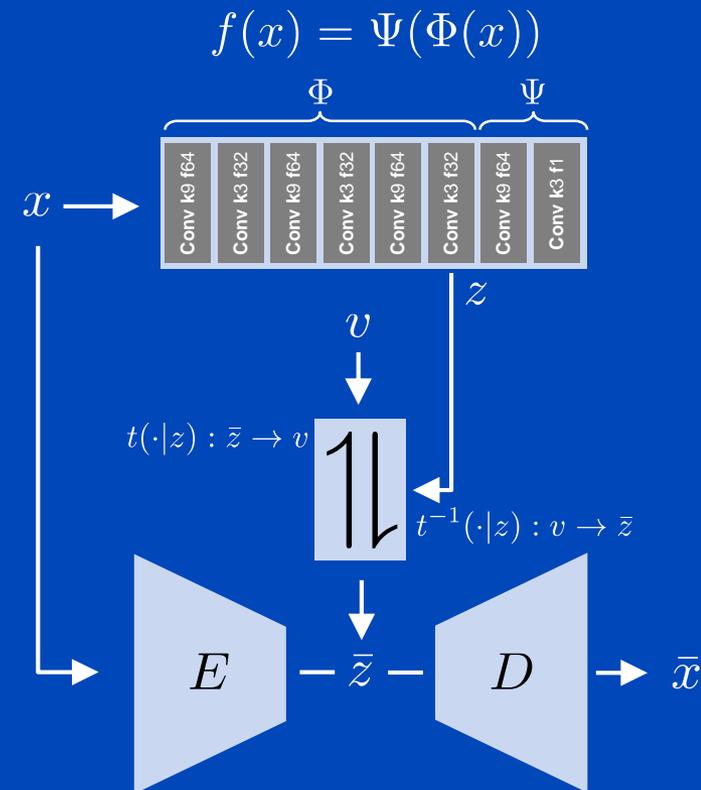
# Methods

## Recovering Invariances

- Our work is based on Rombach et al.<sup>1</sup>
- Given a denoising network  $f(\cdot; \theta)$  we can analyze internal latent representations  $z$  by decomposing  $f(x) = \Psi(z) = \Psi(\Phi(x))$
- To reconstruct which information of  $x$  is captured in  $z$  we train a VAE to learn a complete data representation  $\bar{z} = E(x)$
- To improve reconstruction quality,  $G = D \circ E$  is trained together with critic  $C$  as a Wasserstein GAN

$$\mathcal{L}(E, D) = \mathbb{E}_{\epsilon \sim \mathcal{N}(\epsilon, 0, 1)} \left[ -C(\bar{x}) + \frac{1}{2} \sum_i^{N_{\bar{z}}} \mu_i^2 + \sigma_i^2 - \log(\sigma_i^2) \right]$$

- Train  $G$  on  $128 \times 128$  px<sup>2</sup> patches
- A similar VAE can be trained to learn a complete data representation of high-dose images  $y$



# Methods

## Recovering Invariances

- Disentangle information captured in  $z$  and invariances  $v$  by learning a mapping

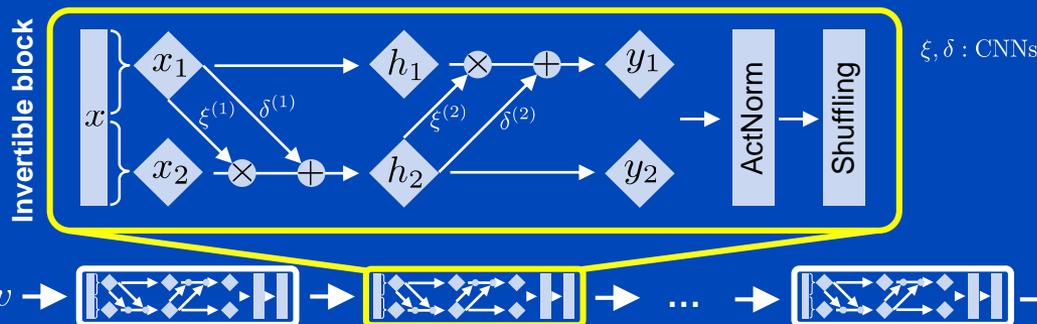
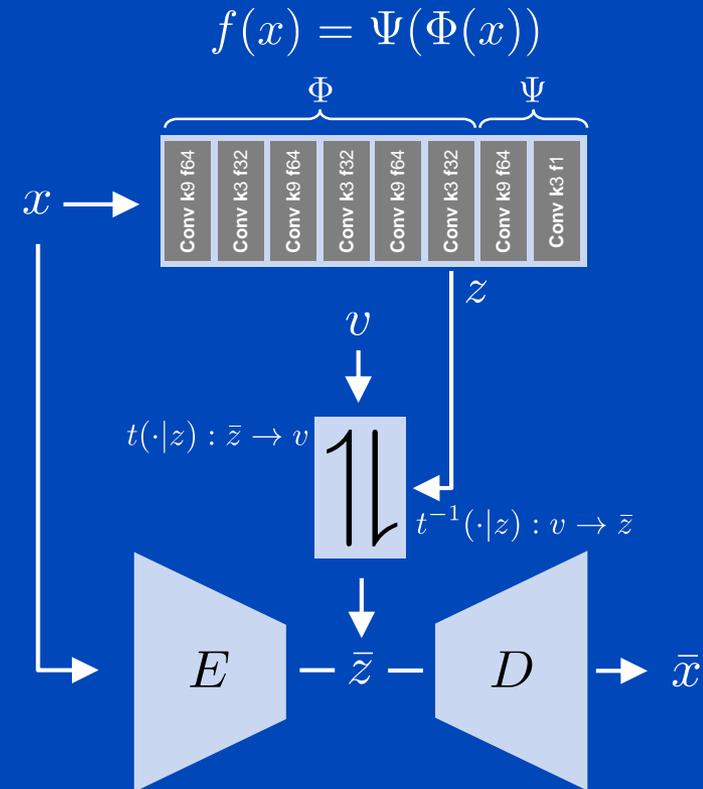
$$t(\cdot|z) : \bar{z} \rightarrow v = t(\bar{z}|z), \quad p(v) = \mathcal{N}(v|0, 1)$$

- $t(\cdot|z)$  is realized by a conditional **invertible neural network<sup>1</sup>** (cINN)

- Generate new  $\bar{z}$  by sampling  $v \sim p(v)$  and then applying the inverse mapping

$$t^{-1}(\cdot|z) : v \rightarrow \bar{z} = t^{-1}(v|z)$$

- Generate new images that only vary in their realization of invariances by applying the decoder  $\bar{x} = D(t^{-1}(v|z))$

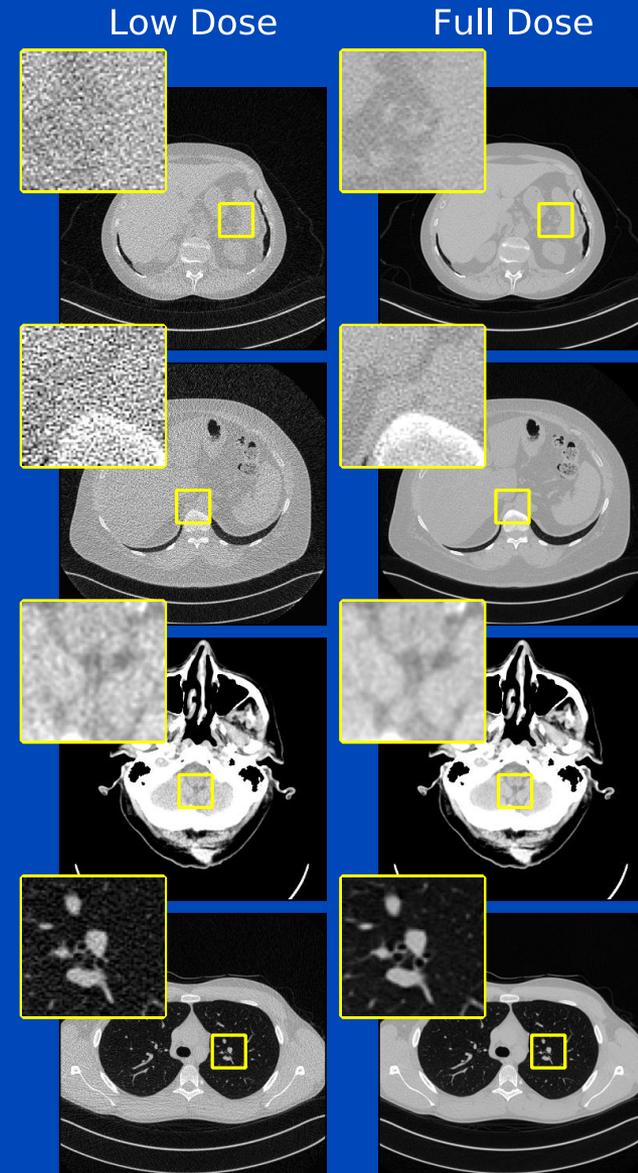


<sup>1</sup>Kingma, Durk P, and Prafulla Dhariwal. "Glow: Generative Flow with Invertible 1x1 Convolutions." NeurIPS, Vol. 31,2018.

# Methods

## Dataset

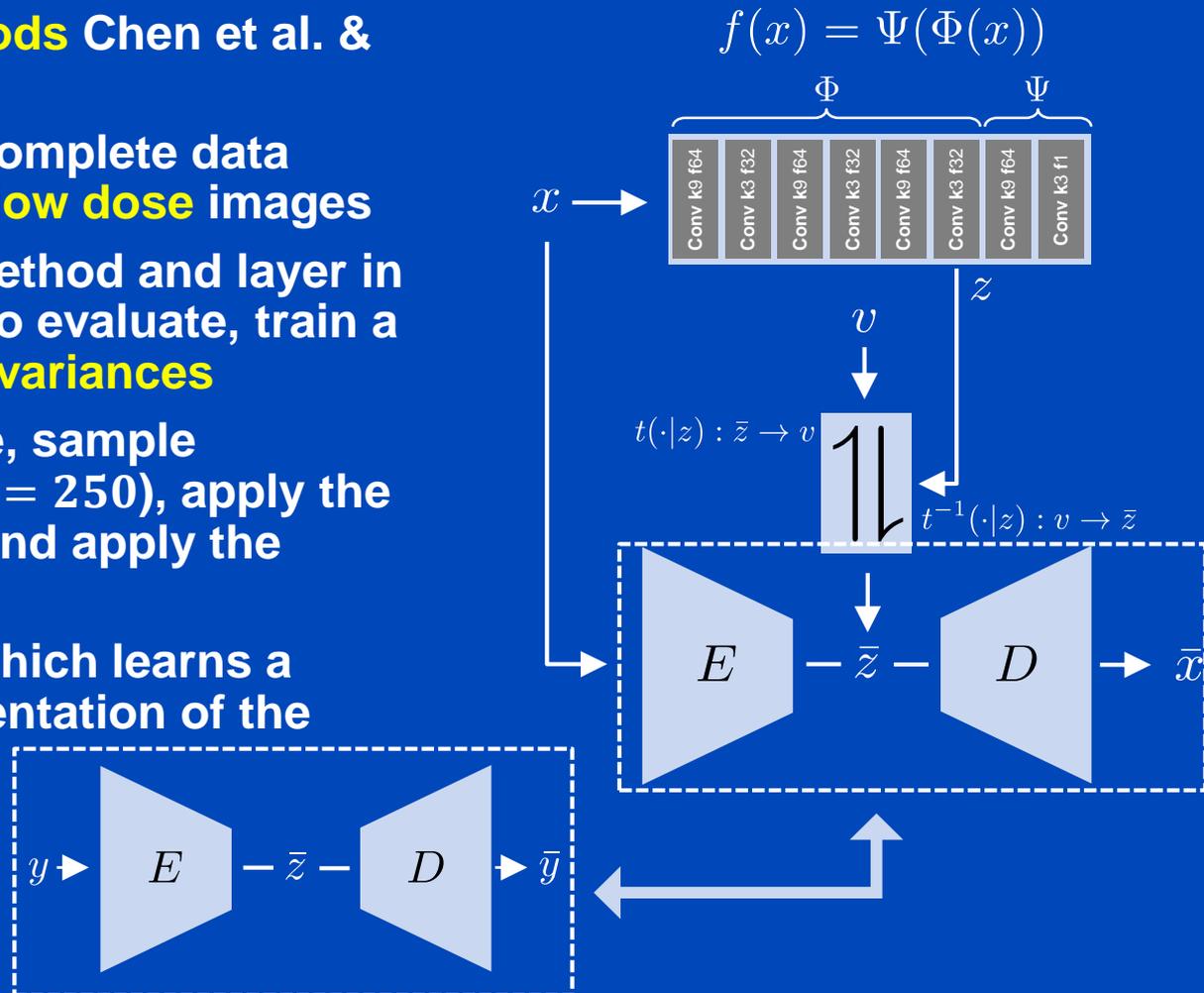
- **Low Dose CT Image and Projection Dataset<sup>1</sup>**
  - 50 {head, chest, abdomen} scans
  - Reconstructions of size  $512 \times 512$  px<sup>2</sup>
  - Acquired with SOMATOM Definition Flash
  - For each scan, simulated low dose acquisitions are available (25% dose for abdomen/head, 10% for chest)
- Use weighted sampling scheme, such that slices from each patient were sampled with equal probability
- Train/validate/test each denoising method and our invariance reconstruction method on the same data splits  
→ Comparable results between different methods



# Methods

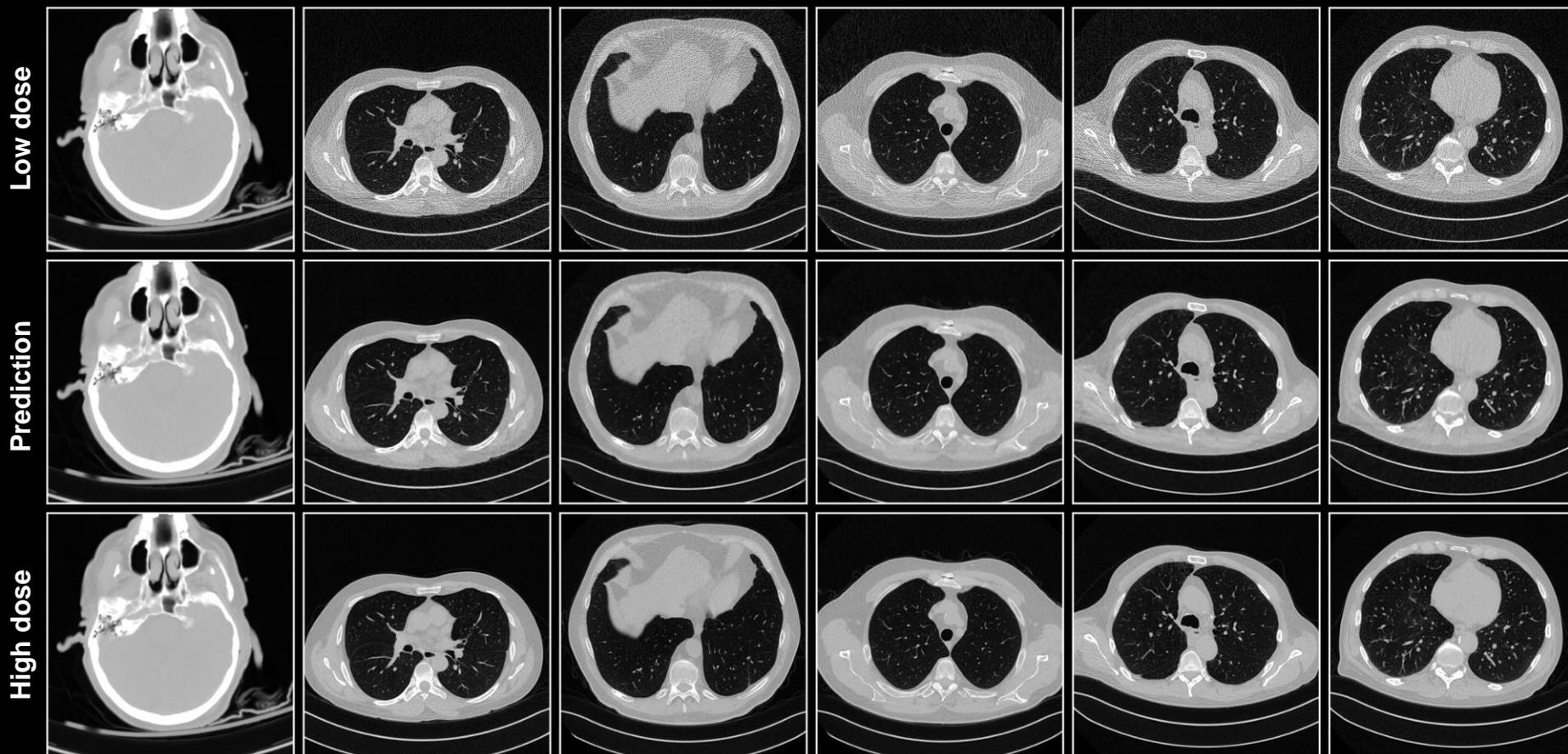
## Summary

1. **Train denoising methods** Chen et al. & Yang et al.
2. Train VAE to learn a complete data representation of the **low dose** images
3. For each denoising method and layer in the network we wish to evaluate, train a **ciNN to recover the invariances**
4. For a given test image, sample  $N$  invariances (here  $N = 250$ ), apply the inverse mapping  $t^{-1}$  and apply the pretrained decoder.
5. Train a second VAE which learns a complete data representation of the **high dose** images



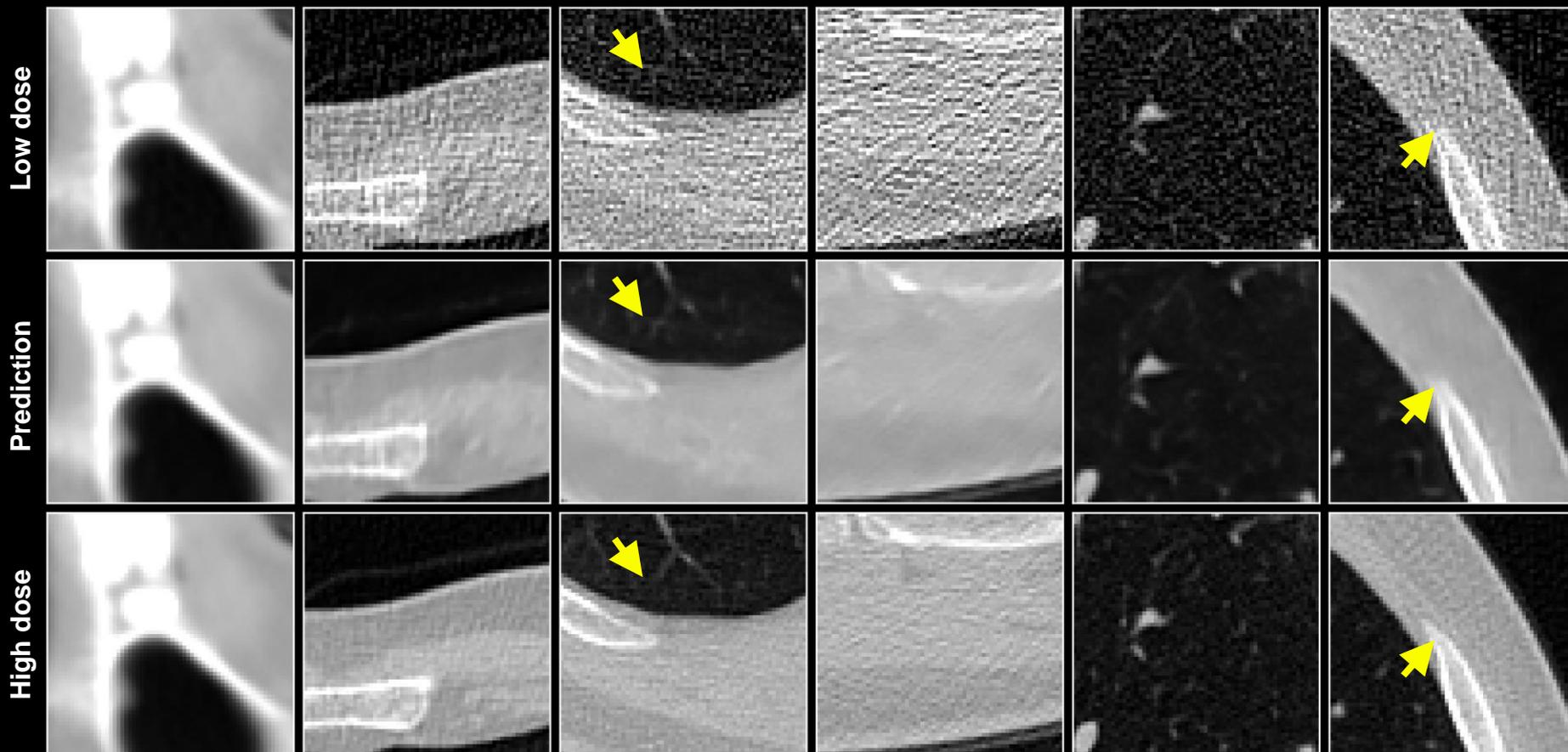
# Results

Denoising (Chen et al.)  $f = \Psi \circ \Phi$



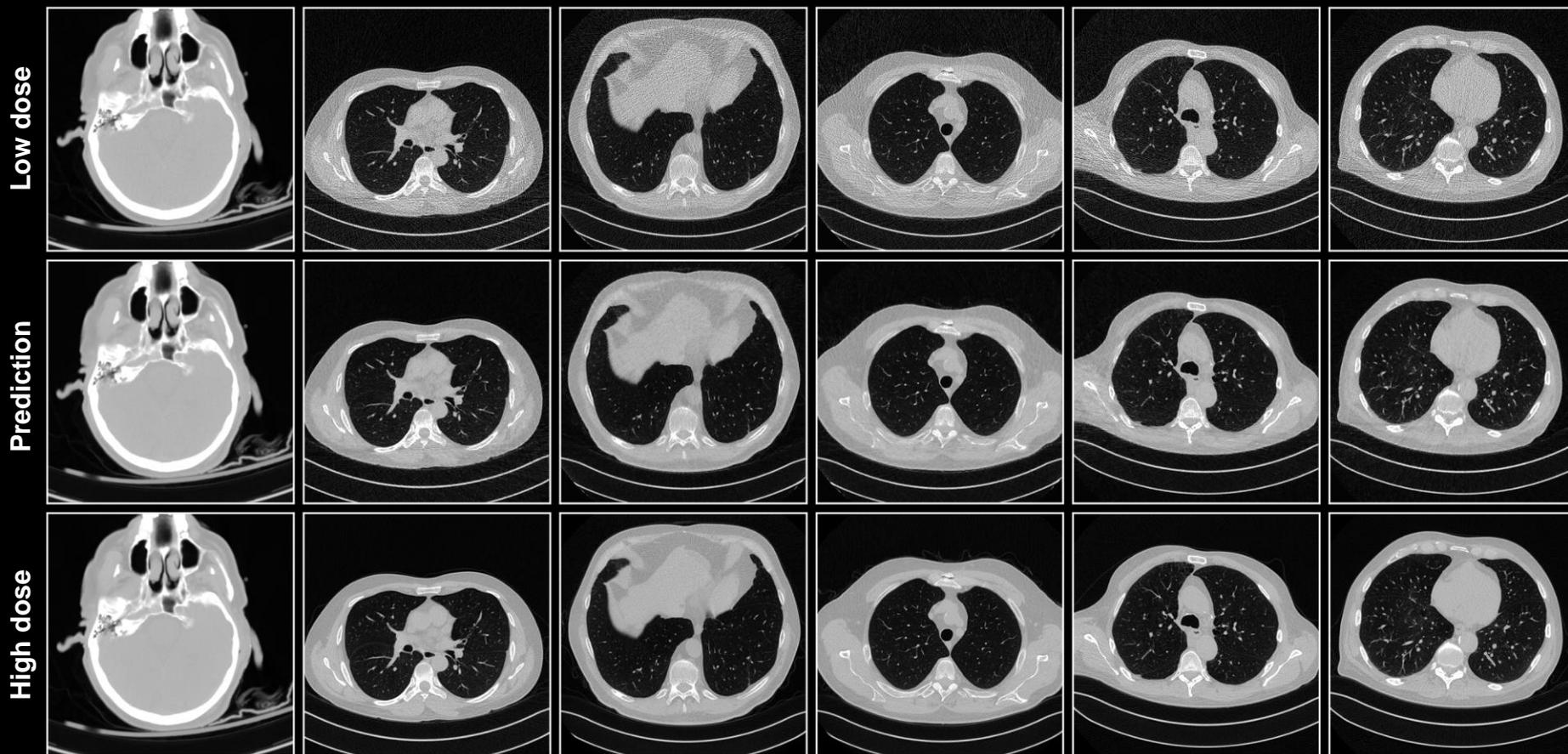
# Results

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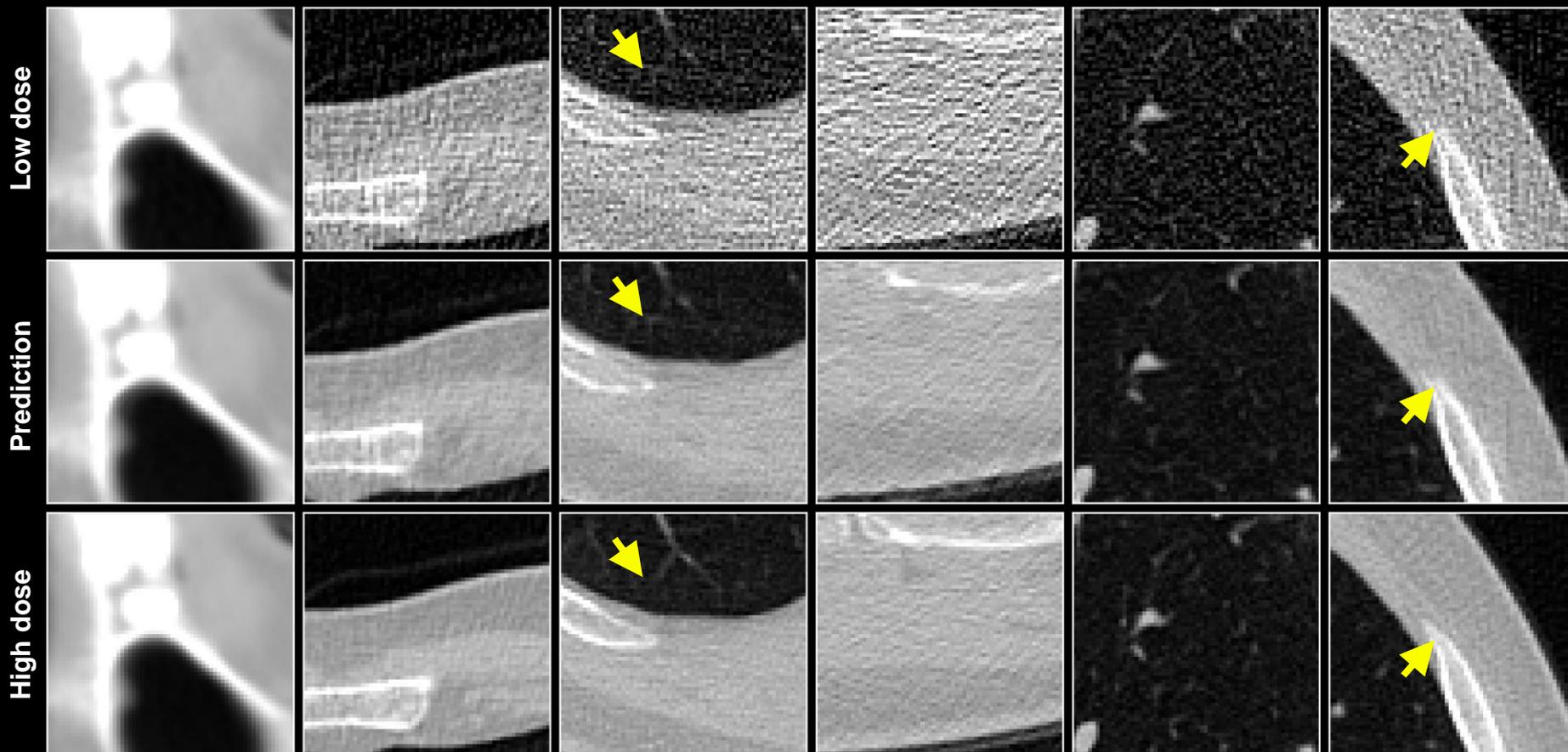
# Results

Denoising (Yang et al.)  $f = \Psi \circ \Phi$



# Results

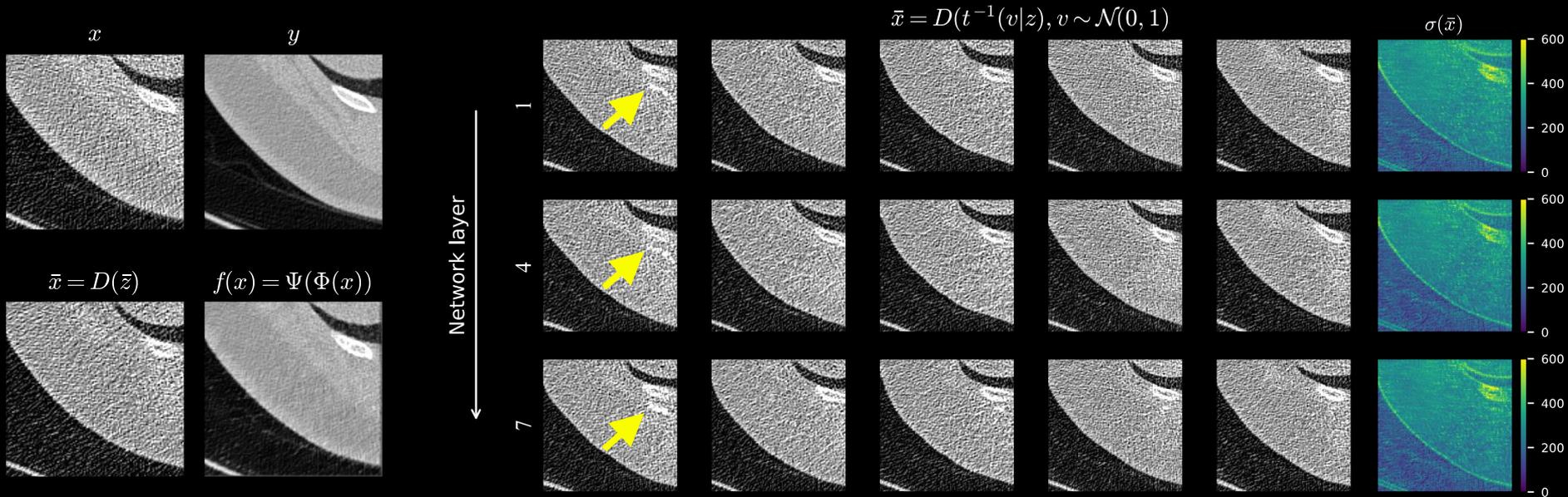
Denoising (Yang et al.)  $f = \Psi \circ \Phi$



- 1 Conv k3 f32
- 2 Conv k3 f32
- 3 Conv k3 f32
- 4 Conv k3 f32
- 5 Conv k3 f32
- 6 Conv k3 f32
- 7 Conv k3 f32
- 8 Conv k3 f11

# Results

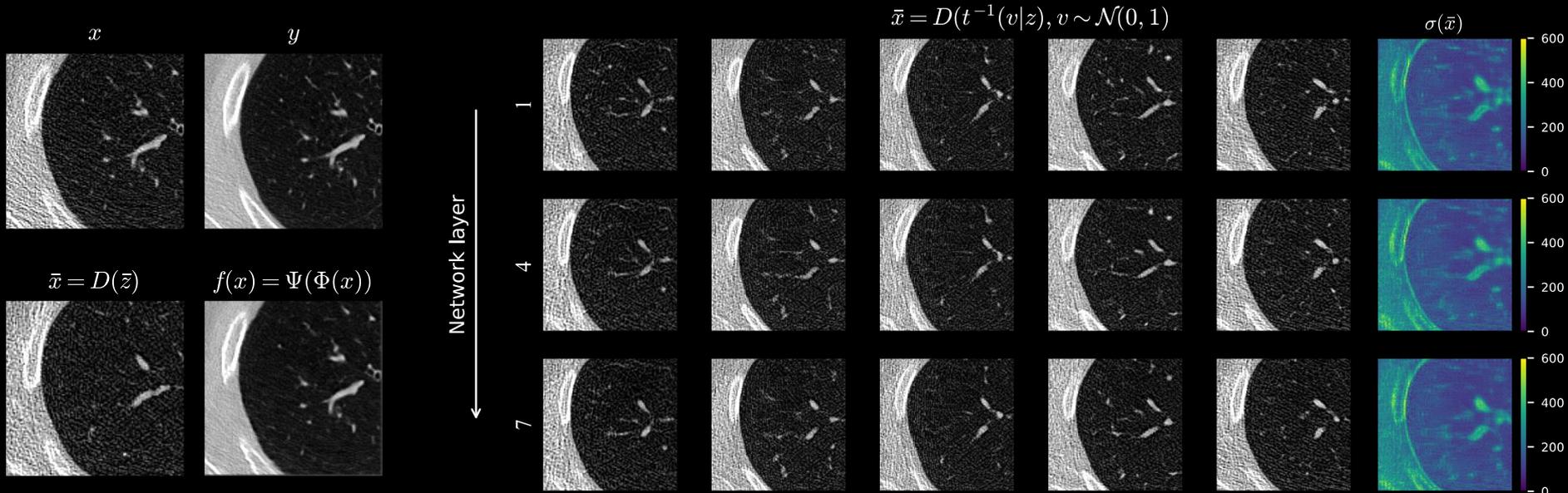
## Sampling Invariances (Yang et al.)



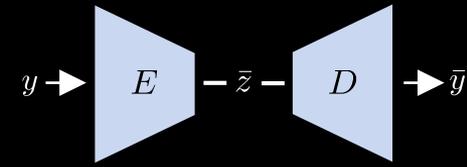
- 1 Conv k3 f32
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- 5 Conv k3 f32
- 6 Conv k3 f32
- 7 Conv k3 f32
- 8 Conv k3 f11

# Results

## Sampling Invariances (Yang et al.)

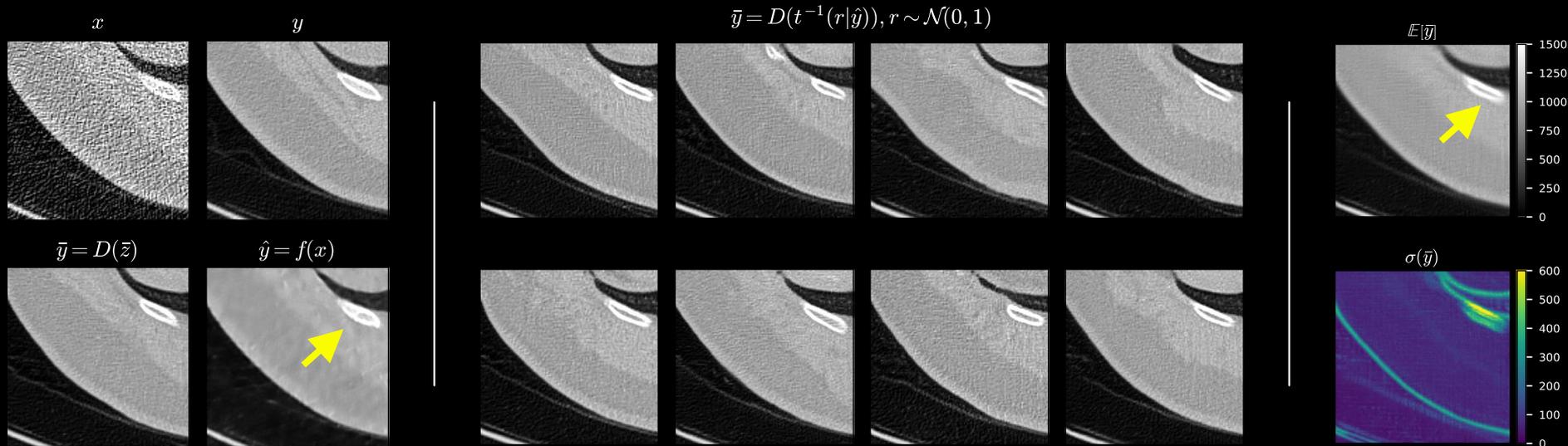


|             |             |            |
|-------------|-------------|------------|
| 1           | 3           | 5          |
| Conv k9 f64 | Conv k3 f32 | Conv k3 f1 |



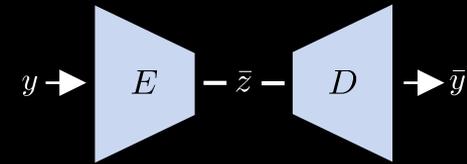
# Results

## Sampling Invariances in Target Domain (Chen et al.)

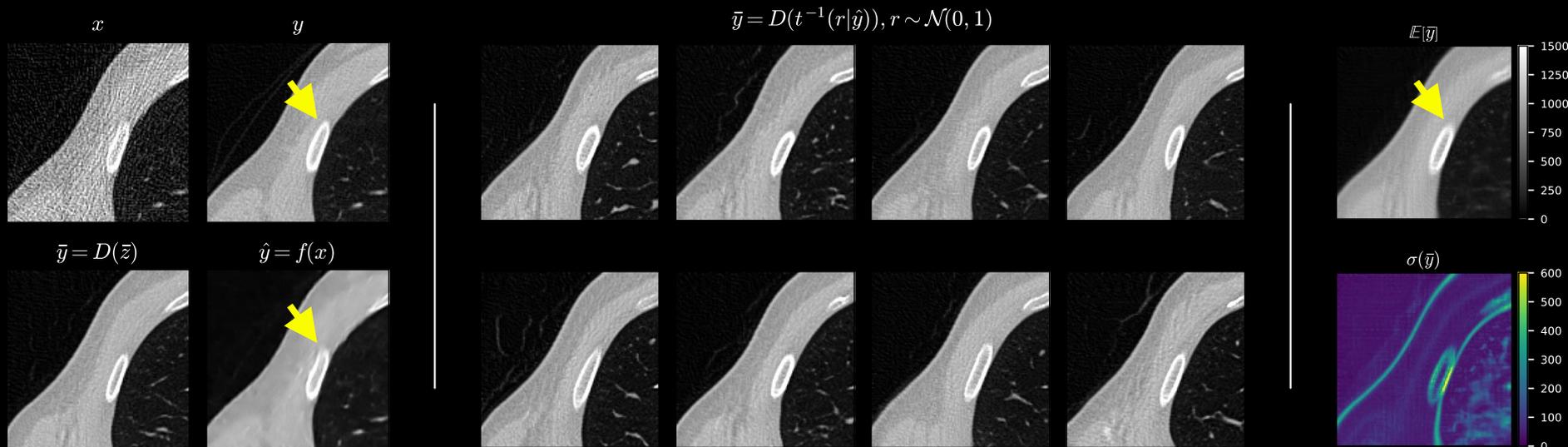


|             |             |            |
|-------------|-------------|------------|
| 1           | 3           | 5          |
| Conv k9 f64 | Conv k3 f32 | Conv k3 f1 |

# Results



## Sampling Invariances in Target Domain (Chen et al.)



# Conclusion & Outlook

## Conclusion

- Both denoising networks perform similar as reported in their respective papers
- Yang et al. produces more realistic results compared to Chen et al. due to training in an adversarial setting
- Both denoising methods are invariant to some anatomical features to some extent
- Incomplete data representation learned by the VAE may explain some of the invariances

## Outlook

- Improve interpretability by
  - Improving the embedding  $\bar{z}$
  - Mapping sampled invariance images  $\bar{x} = D(t^{-1}(v|z))$  to semantically meaningful space
- Minimize “undesired” invariances through a finetuning of the pretrained denoising methods

# Thank You!



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**Job opportunities through DKFZ's international Fellowship programs  
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