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Accurate Reconstruction of X-Ray Spectra in CT from Simple Transmission Measurements

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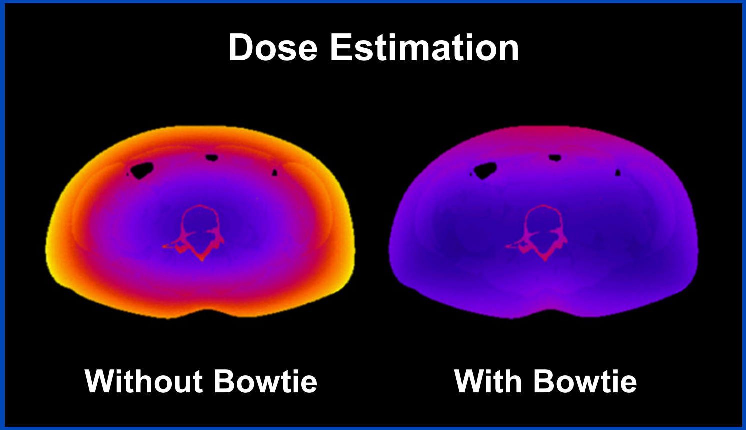
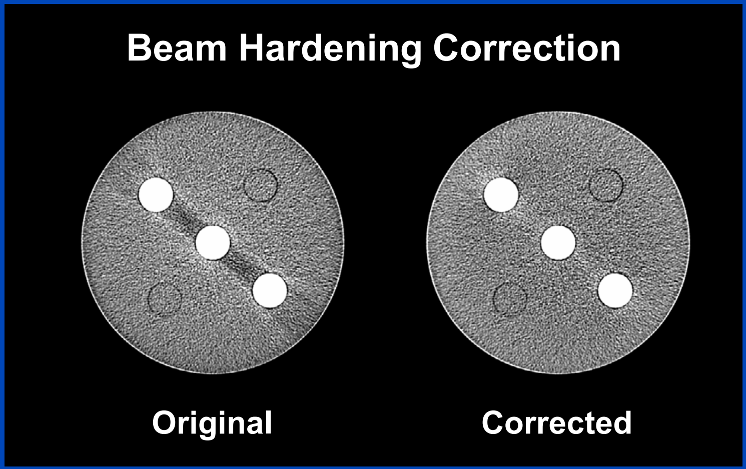
DEUTSCHES KREBSFORSCHUNGSZENTRUM IN DER HELMHOLTZ-GEMEINSCHAFT

Introduction

- CT applications that require accurate knowledge of the emitted or detected spectrum:
  - Organ dose estimation
  - Beam hardening correction
  - Dual energy decomposition
  - K-edge imaging
  - Quantitative perfusion measurements
  - …
- Existing methods:
  - Semi-analytic models
  - Monte-Carlo simulation
  - Spectroscopy
  - Compton scattering
  - Transmission measurements (direct, simple, no extra hardware)
  - …





Materials and Methods

Spectrum Reconstruction from Transmission Measurements

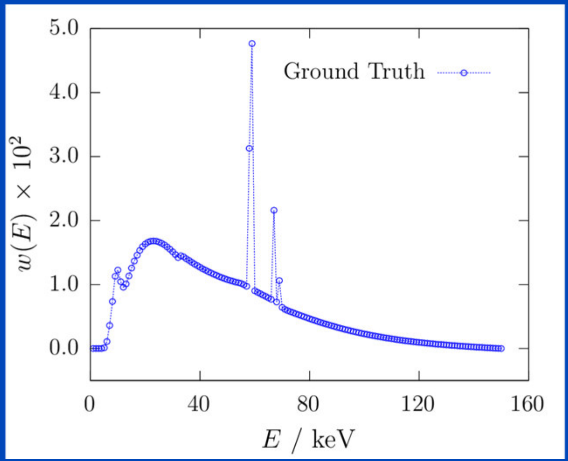
- **Lambert-Beer law:**

$$\tau_m = \frac{N_m}{N_0} = \sum_{b=1}^{B} e^{-\mu_{mb} d_m} w_b$$

- **Problem:**

  "Given *τ* for different (known) combinations of *μ*(E) and *d*, reconstruct w(E)."

- **Methods:**
  - Few parameter modelling
  - Neural networks
  - Expectation maximization (EM)
  - Truncated singular value decomposition (TSVD)
  - New: PTSVD



Tucker et al. - Semi-empirical Model for Generating Tungsten Target X-ray Spectra (1989)

Materials and Methods

Truncated Singular Value Decomposition (TSVD)

- **Discretized Lambert-Beer law in matrix notation:**

$$\tau_m = \sum_{b=1}^{B} a_{mb}\, w_b \longrightarrow \boldsymbol{\tau} = \boldsymbol{A} \cdot \boldsymbol{w}$$

- **Minimize the least square difference**

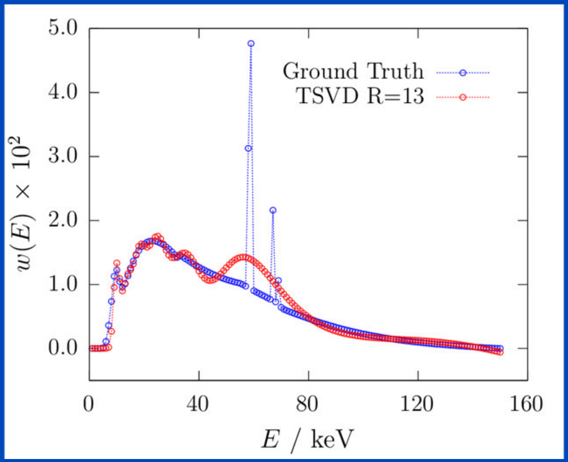
$$\boldsymbol{w} = \arg\min_{\boldsymbol{w}} \|\boldsymbol{A} \cdot \boldsymbol{w} - \boldsymbol{\tau}\|_2^2 \longrightarrow \boldsymbol{w} = \boldsymbol{A}^{+} \cdot \boldsymbol{\tau}$$

- **Calculation of the pseudo-inverse A^+**
  - **Decompose A into orthonormal basis with help of SVD:**

$$\boldsymbol{A} = \sum_{b=1}^{B} \boldsymbol{u}_b \cdot s_b \boldsymbol{v}_b^T$$

  - **Truncate A^+ to the highest R singular values:**

$$\boldsymbol{w} = \sum_{b=1}^{R} \left(\boldsymbol{v}_b \cdot \frac{\boldsymbol{u}_b^T}{s_b} \right) \cdot \boldsymbol{\tau} \qquad R \leq B$$



Materials and Methods

Prior Truncated Singular Value Decomposition (PTSVD)

- **Minimize the weighted least square difference with help of TSVD to obtain the low frequent solution from range:**

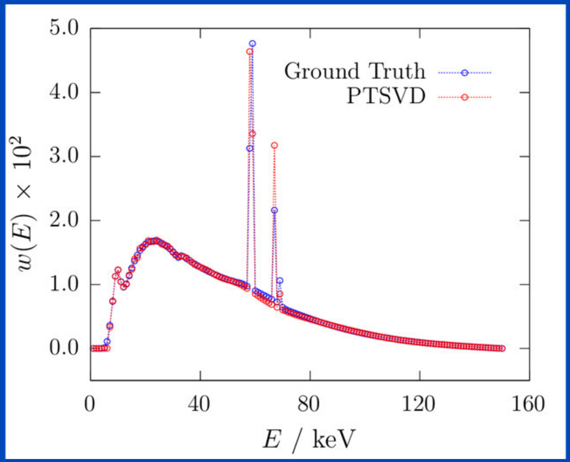
$$\boldsymbol{w}_R = \arg\min_{\boldsymbol{w}} \|\boldsymbol{A}\cdot\boldsymbol{w} - \boldsymbol{\tau}\|_{\boldsymbol{W}}^2 \quad \textbf{with} \quad \boldsymbol{W} = \mathrm{Cov}(\boldsymbol{\tau},\boldsymbol{\tau})^{-1}$$

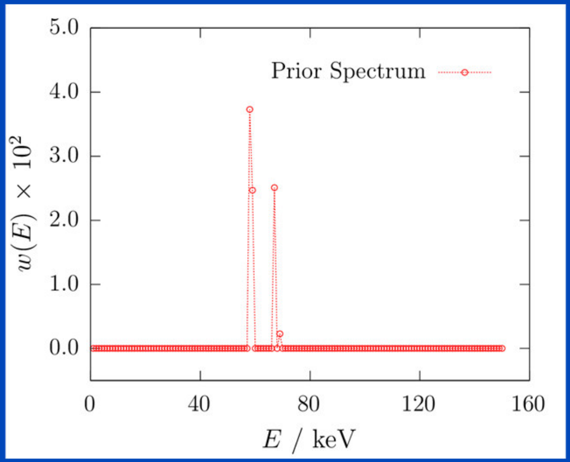
- **Calculate a solution from null space that represents the high frequency components (here: characteristic peaks):**

$$\boldsymbol{w}_{\mathrm{N}} = \sum_{b=R+1}^{B} (\boldsymbol{v}_b^T \cdot \boldsymbol{w}_{\mathrm{H}})\,\boldsymbol{v}_b$$

- **Add the solution from null space to the solution from range:**

$$\boldsymbol{w} = \boldsymbol{w}_R + \boldsymbol{w}_N$$





Materials and Methods

Prior Truncated Singular Value Decomposition (PTSVD)

- **We model the prior spectrum:**

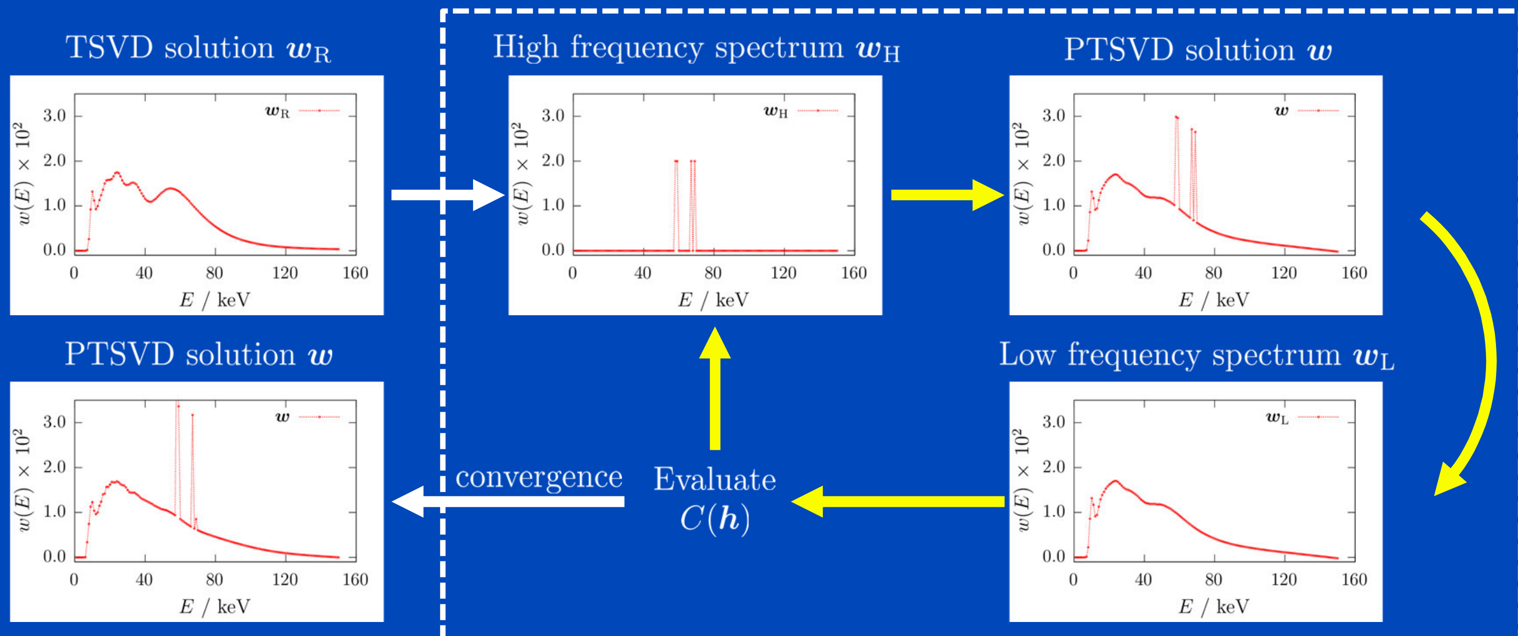
$$\boldsymbol{w}_{\mathrm{H}}(\boldsymbol{h}) = \sum_{p=1}^{P} h_p \, \boldsymbol{e}_p$$

- **Cost function**

$$C(\boldsymbol{h}) = \underbrace{\|\boldsymbol{w}_L(\boldsymbol{h}) \wedge \boldsymbol{0}\|_2^2}_{\text{Non-negativity}} \quad + \quad \lambda \underbrace{\|\boldsymbol{\nabla} \cdot \boldsymbol{w}_{\mathrm{L}}(\boldsymbol{h})\|_2^2}_{\text{Smoothness}}$$

$$\boldsymbol{w}_{\mathrm{L}}(\boldsymbol{h}) = \boldsymbol{w}(\boldsymbol{h}) - \boldsymbol{w}_{\mathrm{H}}(\boldsymbol{h})$$

- **Iteration schema:**



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Materials and Methods

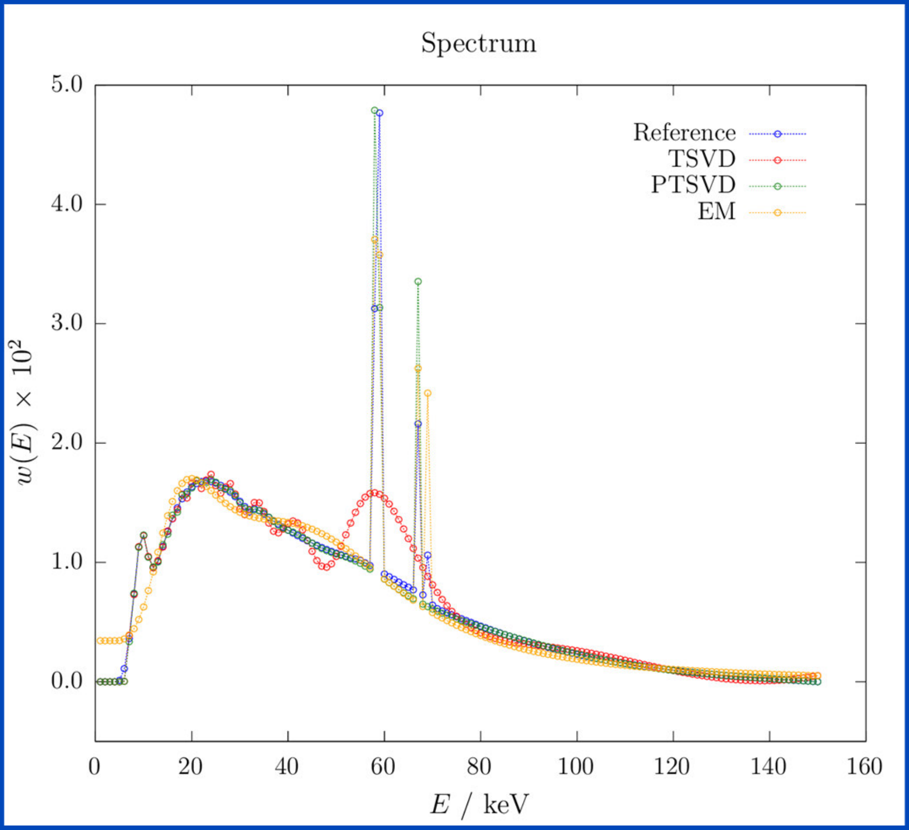
Simulation / Measurement Study

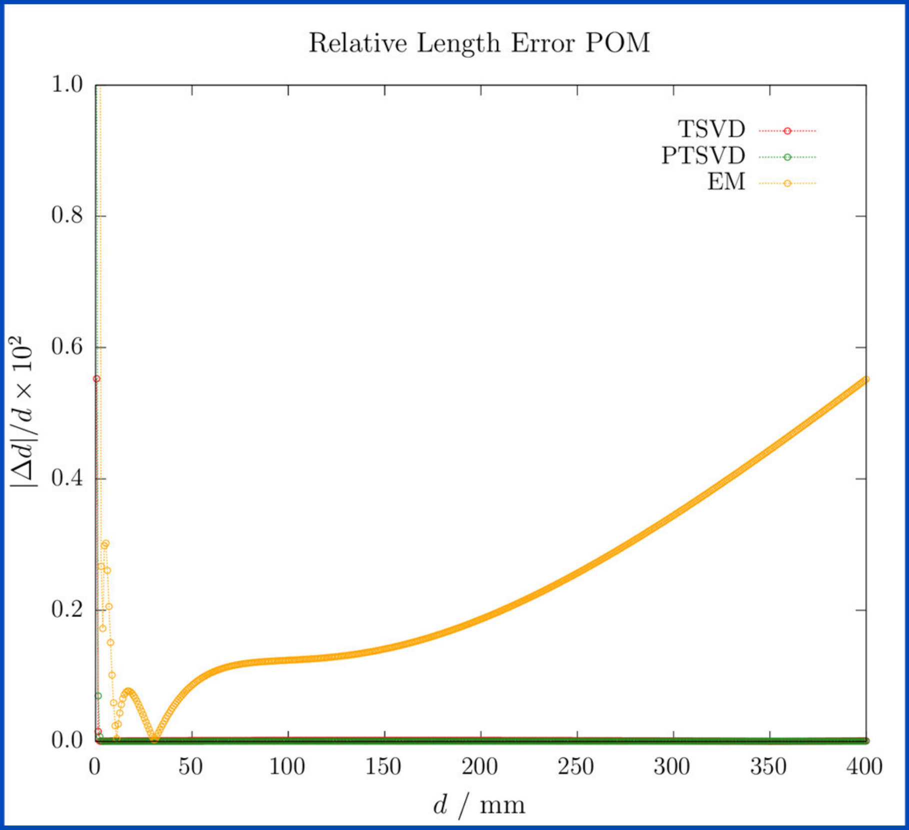
- **Simulation conditions:**
  - 150 kV tungsten target spectrum simulated according to Tucker et al.
  - Spectrum estimation from 28 aluminum (Al) attenuators with lengths ranging from 0.5 mm to 132.5 mm
  - Poisson noise is added to the Al transmission data for varying numbers of incident photons N_0
  - Noiseless simulations of polyoxymethylene (POM) with continuous attenuation length for validation

- **Measurement conditions:**
  - Experimental setup consisting of a 150 kV transmission x-ray tube and a flat detector
  - 28 measurements of Al and POM attenuators with attenuation lengths ranging from 0.5 mm to 132.5 mm
  - Material for spectrum estimation: Al
  - Material for spectrum validation: POM

Results

Noiseless Simulated Data





$$\frac{|\Delta d|}{d} = \frac{|d' - d|}{d}$$

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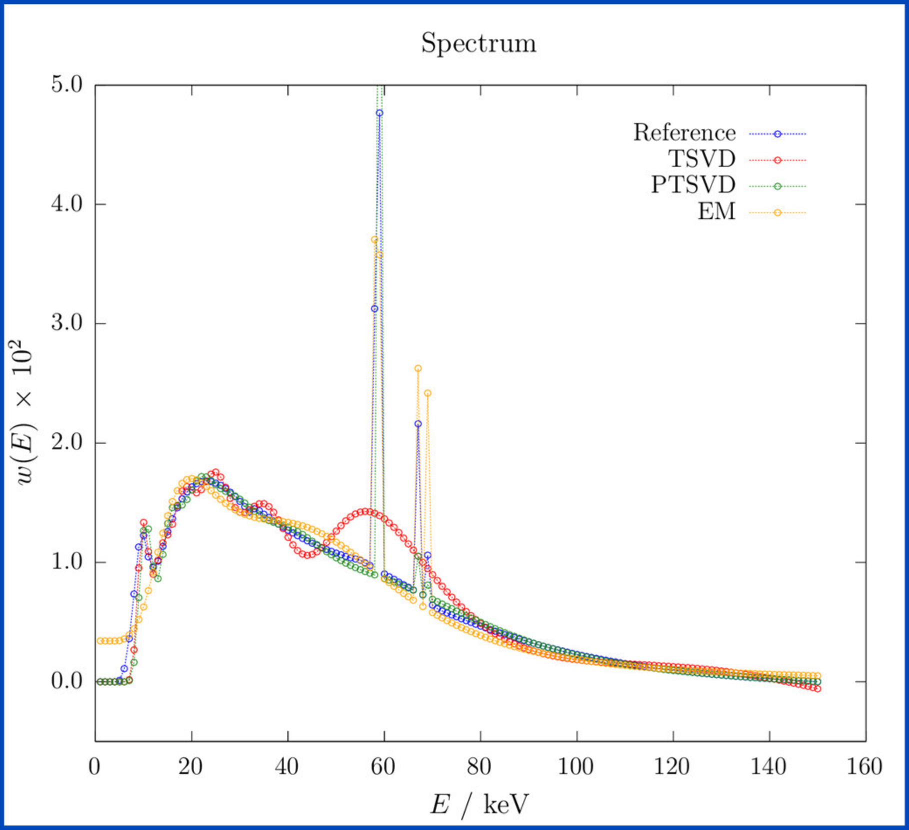
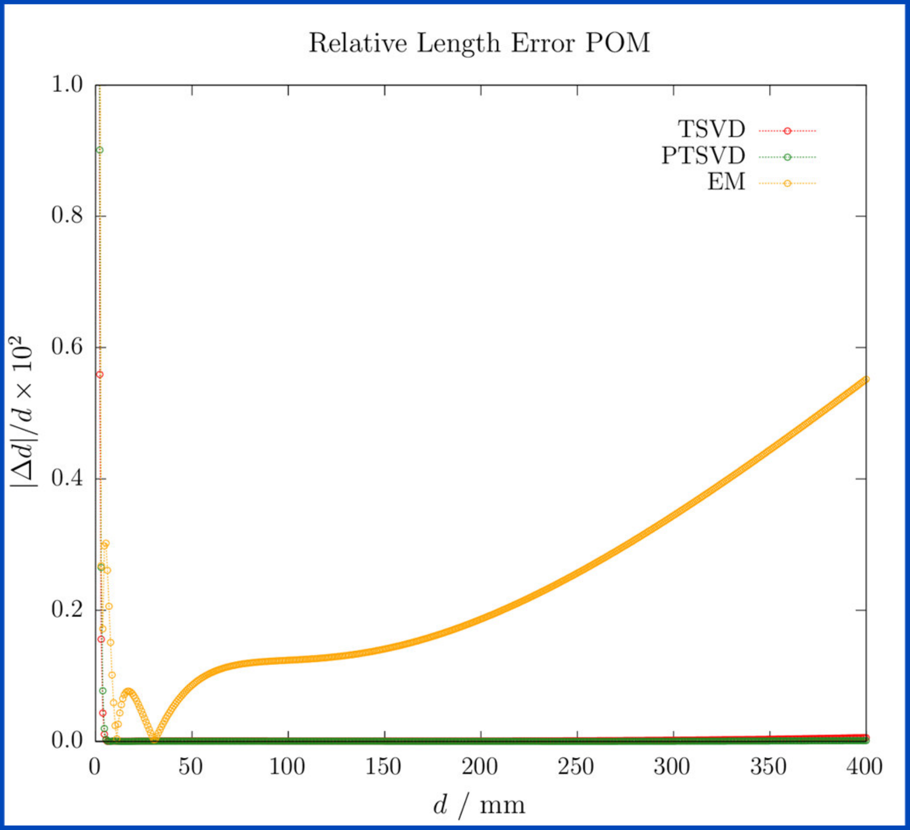
Results

Noisy Simulated Data $N_0 = 1\times10^{12}$





$$\frac{|\Delta d|}{d} = \frac{|d' - d|}{d}$$

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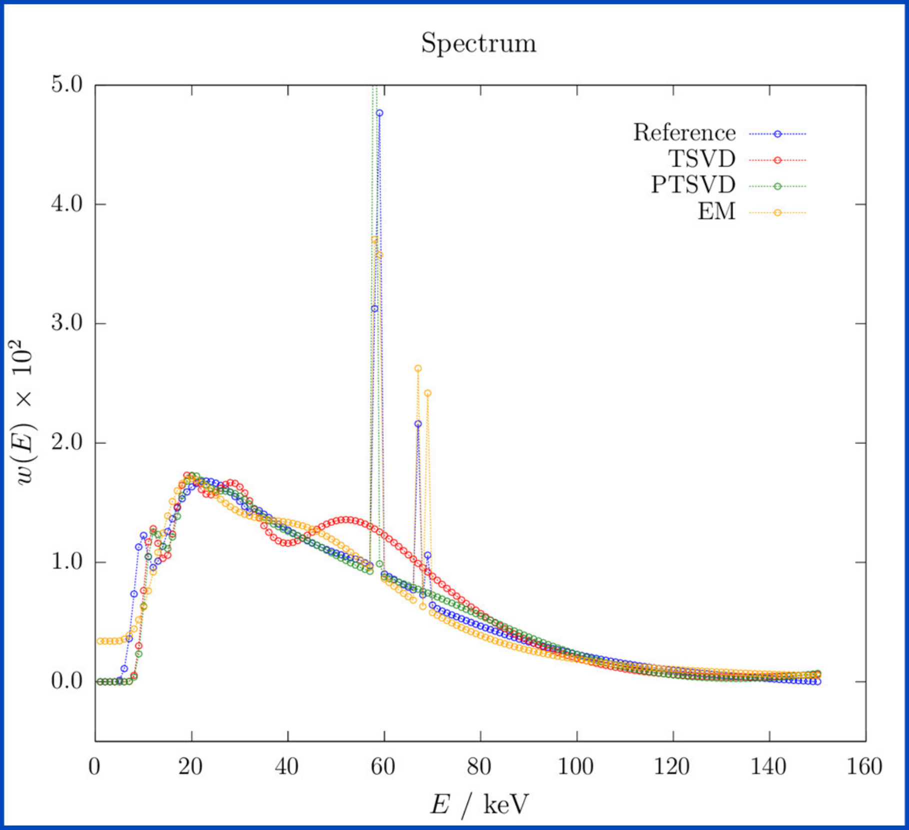
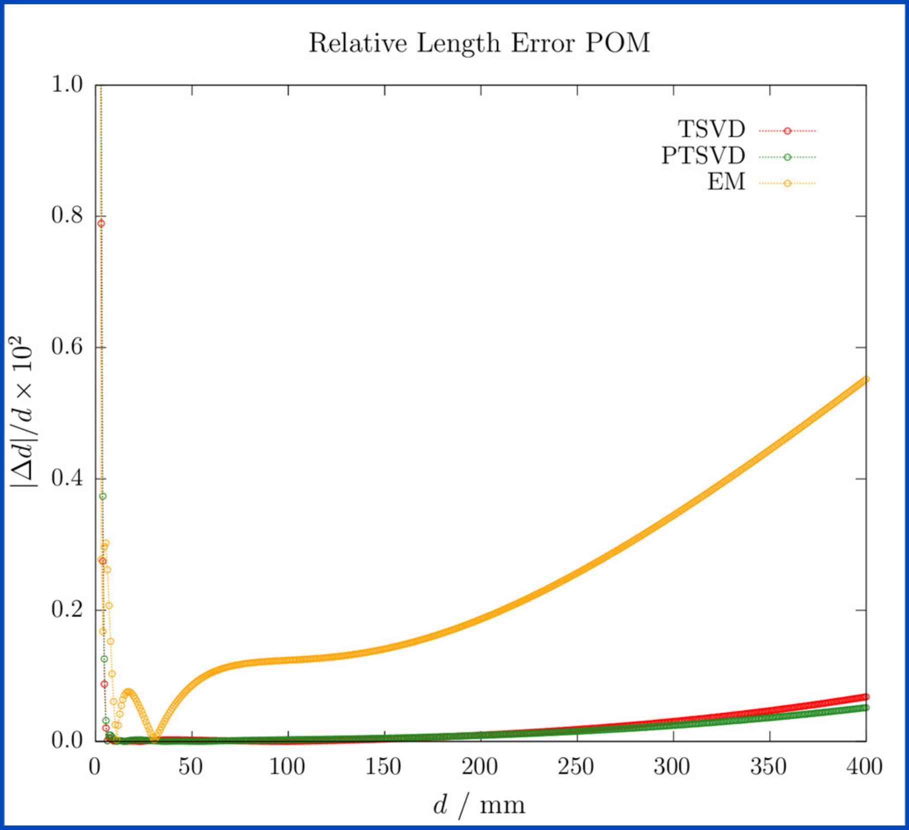
Results

Noisy Simulated Data $N_0 = 1\times10^{10}$





$$\frac{|\Delta d|}{d} = \frac{|d' - d|}{d}$$

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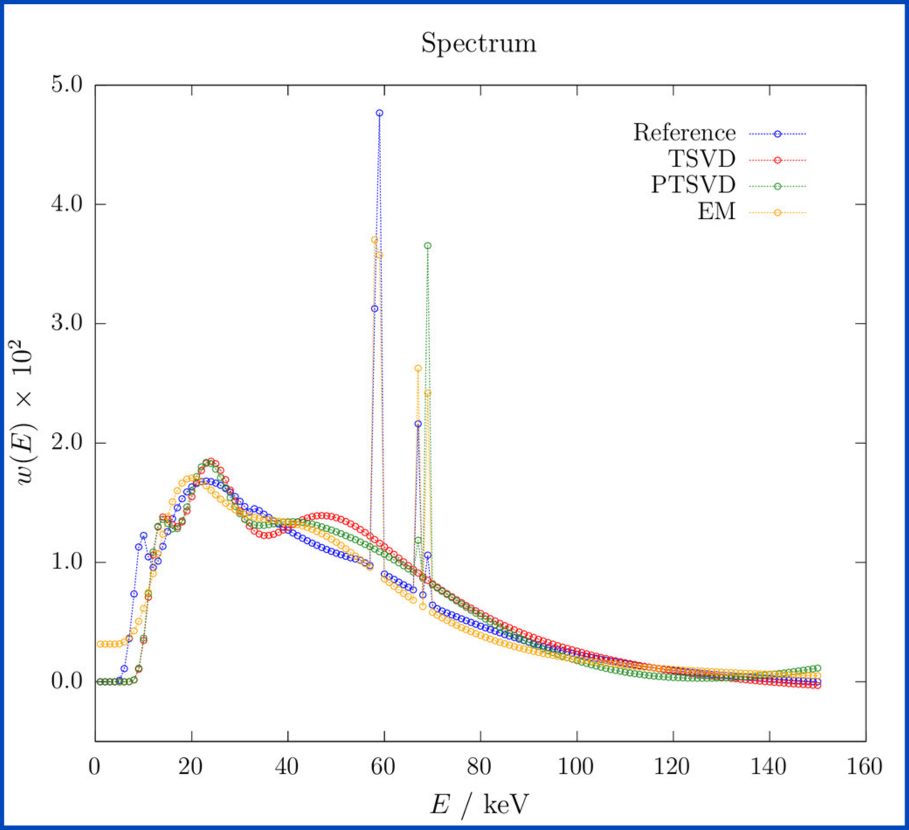
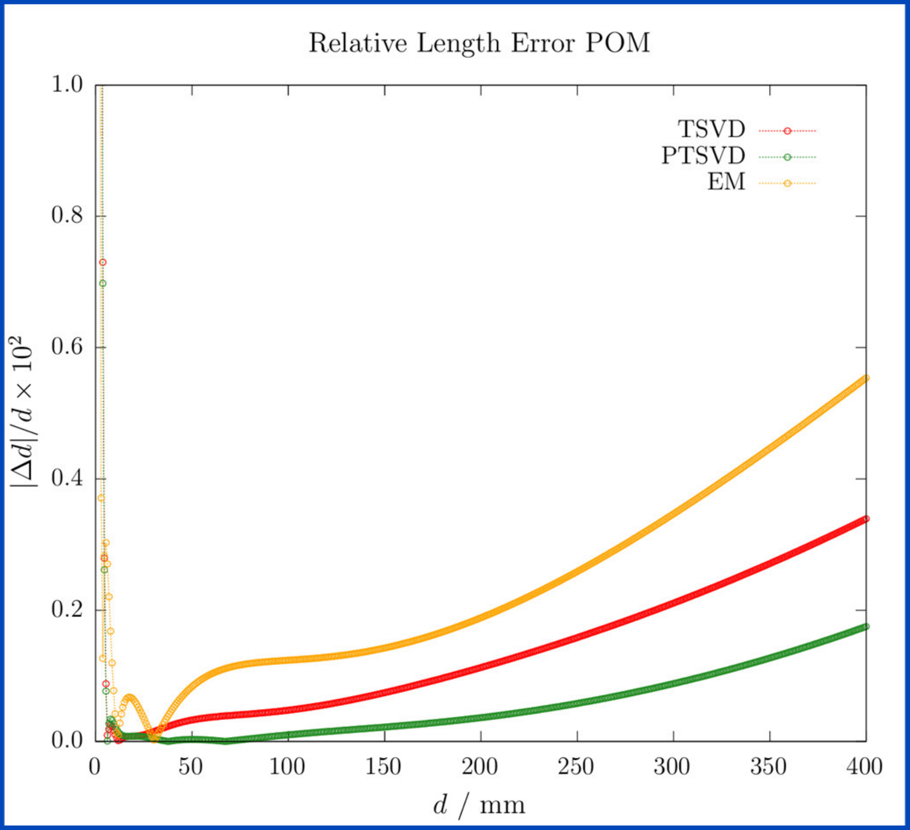
Results
Noisy Simulated Data
$N_0 = 1\times10^8$

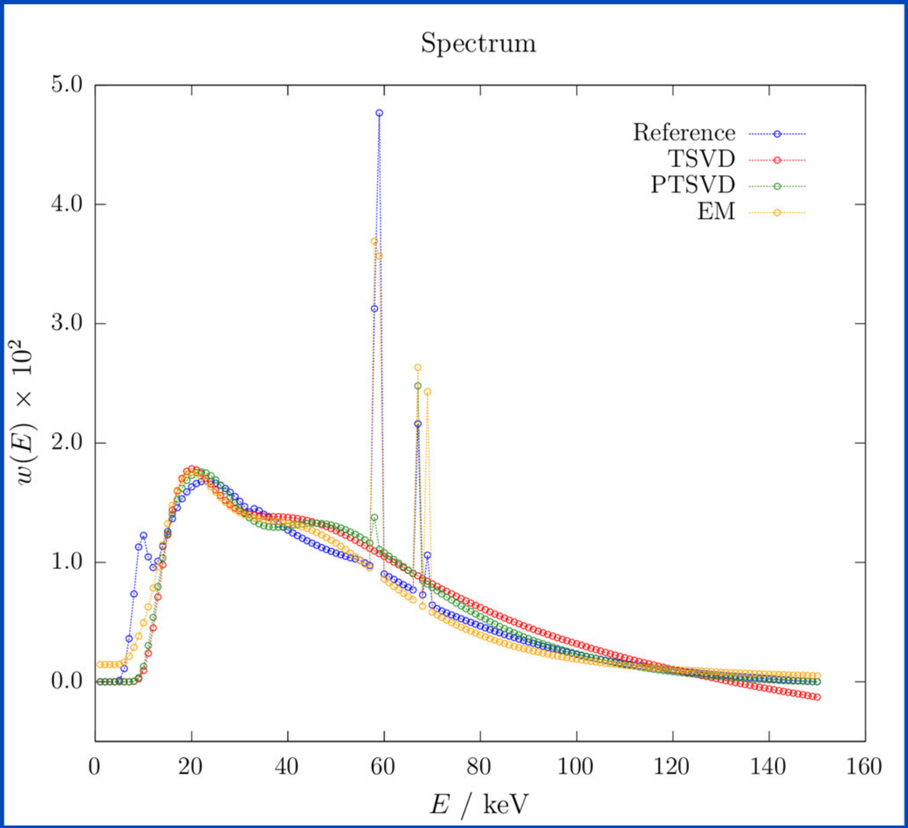


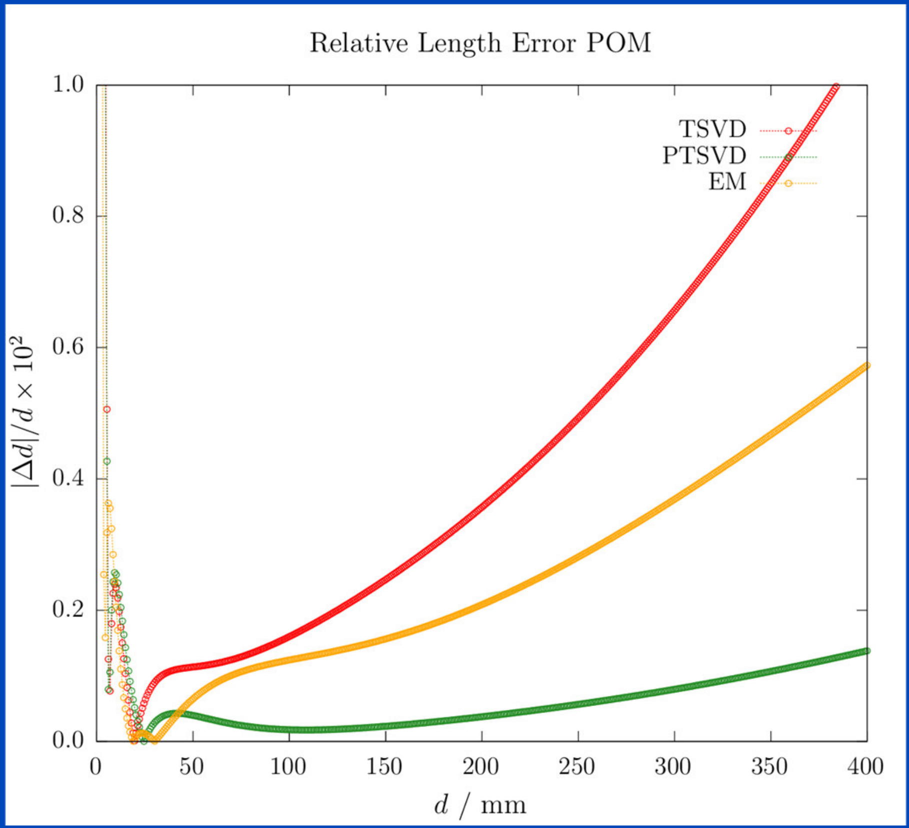


$$\frac{|\Delta d|}{d} = \frac{|d' - d|}{d}$$

Results

Noisy Simulated Data
$N_0 = 1\times10^6$



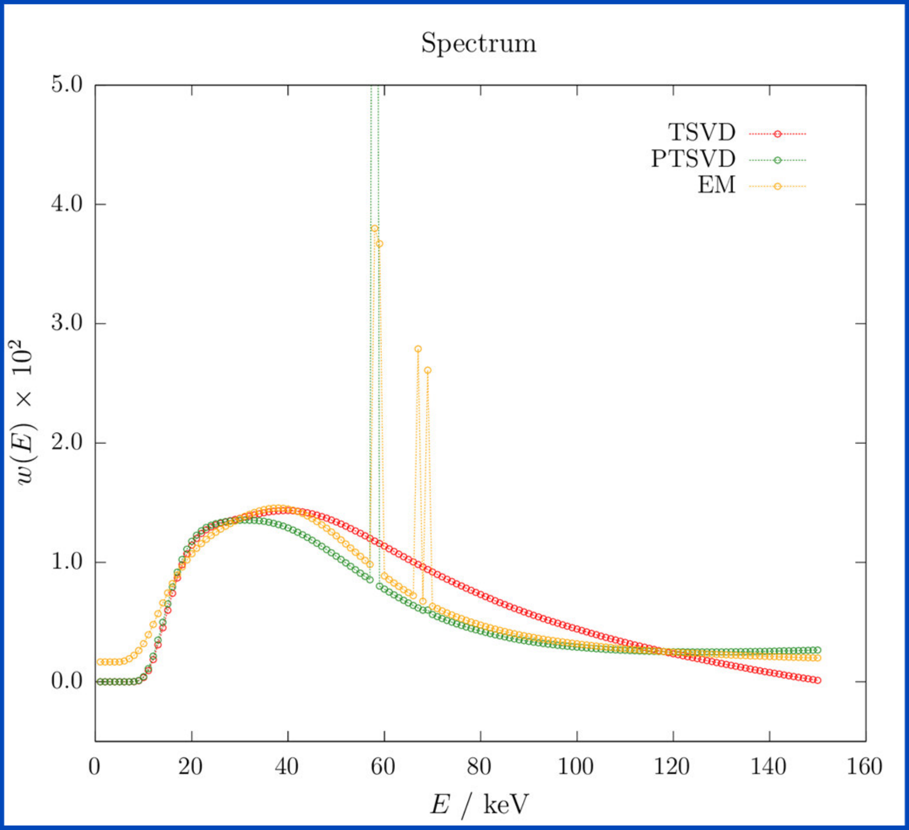


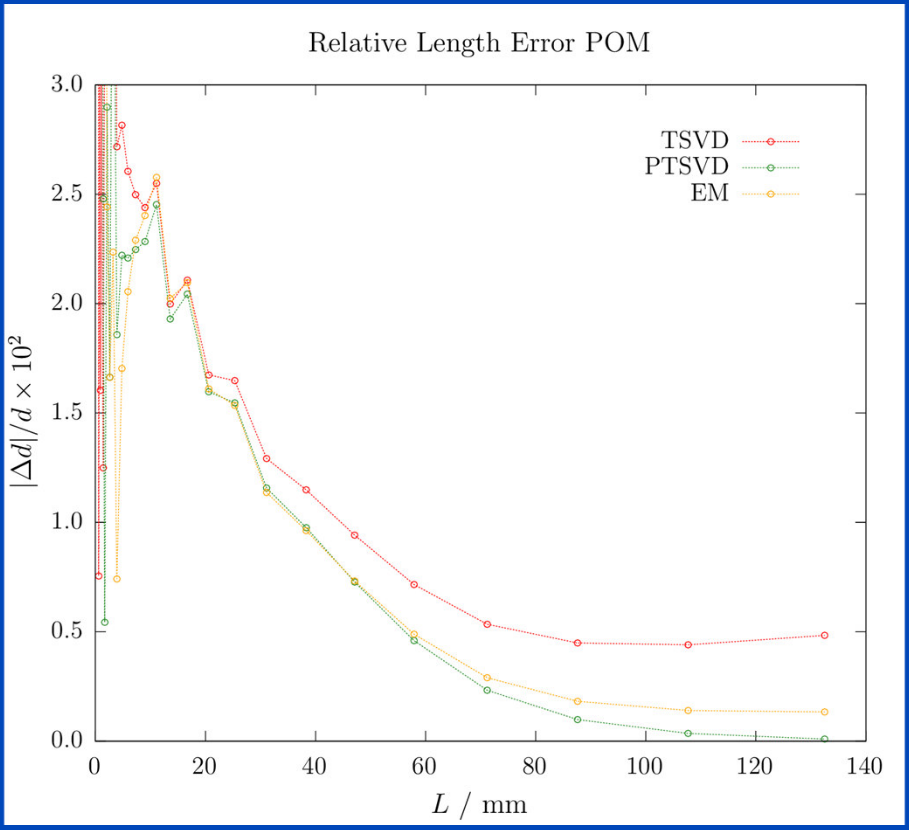
$$\frac{|\Delta d|}{d} = \frac{|d' - d|}{d}$$

Results

Measured Data

$N_0 \approx 1\times 10^{10}$





$$\frac{|\Delta d|}{d} = \frac{|d' - d|}{d}$$

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Conclusion and Discussion

- PTSVD overcomes the limitations of TSVD by incorporating prior information about the statistical nature of the transmission data and about the high frequency components of the spectrum.
- PTSVD is less prone to noise compared to TSVD.
- Simulations show that for accurate transmission data PTSVD leads to smaller length errors compared to EM.
- Effects that limit the accuracy of transmission measurements: quantum noise, electronic noise, scattered radiation, image lag, quantization errors, dynamic range, …

Thank You!

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This presentation will soon be available at www.dkfz.de/ct.

Job opportunities through DKFZ's international PhD or Postdoctoral Fellowship programs (www.dkfz.de), or directly through Marc Kachelrieß (marc.kachelriess@dkfz.de).

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