Alpha Image Reconstruction (AIR)
Optimization and Image Quality Assessment

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Aims

- Increase convergence speed of the AIR[1] algorithm.
- Demonstrate that conventional image quality metrics can be applied to the AIR images.

[1]Hofmann, Sawall, Knaup, and Kachelrieß, "Alpha Image Reconstruction (AIR): A New Iterative CT Image Reconstruction Approach Using Voxel-Wise Alpha Blending", *Med. Phys. 41(6), p. 061914 (14 pages), 2014*

AIR[1]

- AIR = Iterative algorithm that combines multiple, voxel-wise weighted basis images into one new image f_{AIR}

$$f_{\mathrm{AIR}} = \sum_{b=1}^{B} \underbrace{\alpha_b}_{} \circ f_b$$

Contains weighting coefficients for every voxel

α_b = weighting images

f_b = basis images

$\alpha \circ f$ = Hadamard product

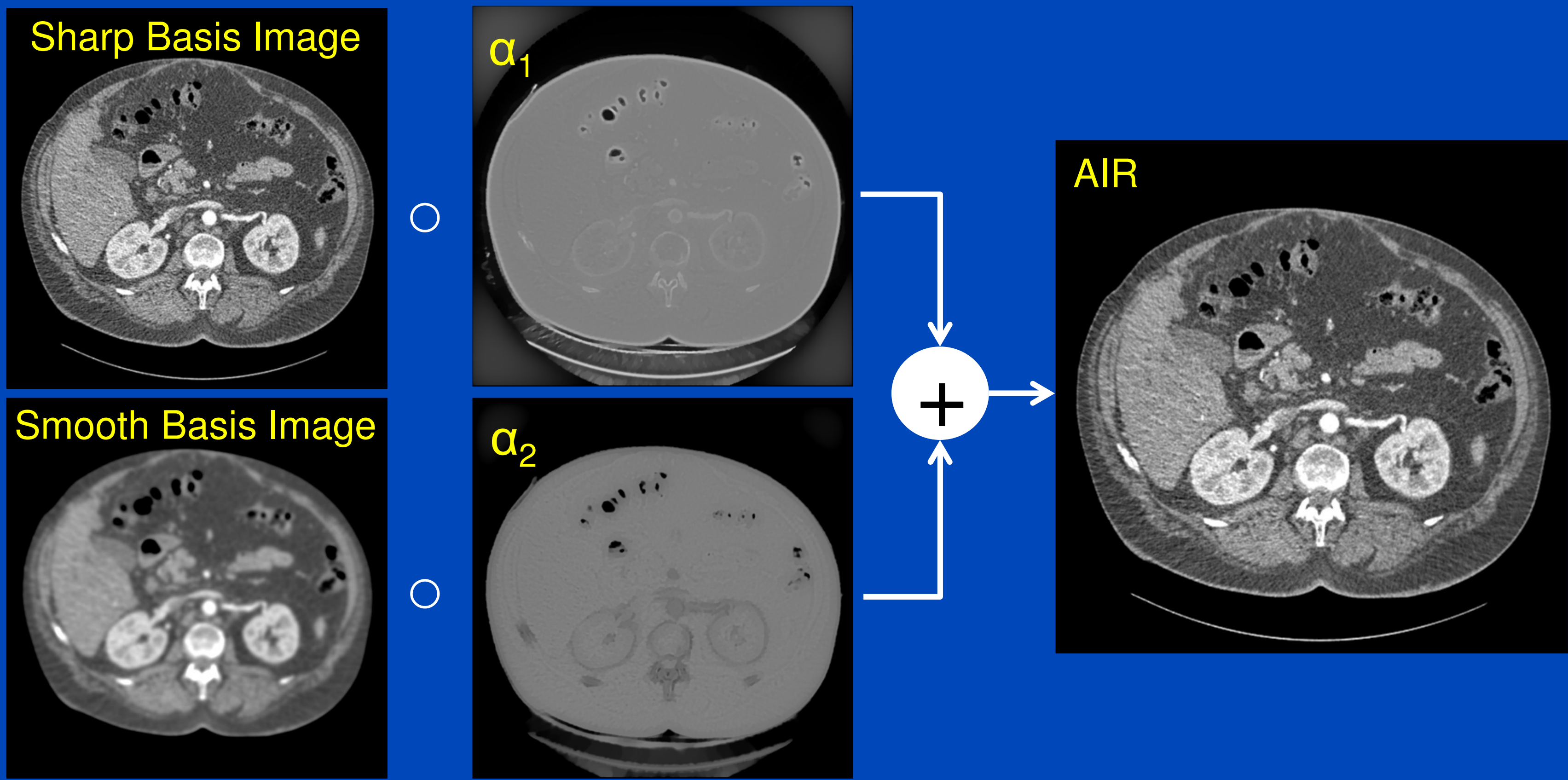
B = number of basis images

- Idea: each basis image has different characteristics such as low noise or high spatial resolution → all characteristics combined in one new image.

[1]Hofmann, Sawall, Knaup, and Kachelrieß, "Alpha Image Reconstruction (AIR): A New Iterative CT Image Reconstruction Approach Using Voxel-Wise Alpha Blending", *Med. Phys. 41(6), p. 061914 (14 pages), 2014*

AIR

- **Basic application: combination of a sharp and smooth FBP.**



Known basis images.

Weighting images to be computed.

AIR

- **AIR minimizes a cost function:**

Rawdata Fidelity — Regularization

$$C(\boldsymbol{\alpha}) = \underbrace{||\mathbf{X}\left(\sum_{b=1}^{B}\boldsymbol{\alpha}_b \circ \boldsymbol{f}_b\right) - \boldsymbol{p}||_{\boldsymbol{W}}^2}_{\textbf{convex}} + \beta \underbrace{\sum_{b=1}^{B} TV(\boldsymbol{\alpha}_b)}_{\text{TV regularization, } \textbf{convex}} + \gamma \underbrace{\sum_{b=1}^{B} ||\boldsymbol{\alpha}_b - \boldsymbol{d}_b||_2^2}_{\text{Penalizes deviations from default images } \mathbf{d}_b\text{, strictly convex.}}$$

- **Gradient descent approach:**

$$\boldsymbol{\alpha}_b^{\nu+1} = \boldsymbol{\alpha}_b^{\nu} - \lambda \boldsymbol{\nabla}_{\alpha_b} C(\boldsymbol{\alpha}_b^{\nu})$$

$$\boldsymbol{\nabla} C(\boldsymbol{\alpha}_b) = \boldsymbol{f}_b \circ \left(\mathbf{X}^T \boldsymbol{W}\left(\mathbf{X}\left(\sum_{b=1}^{B}\boldsymbol{\alpha}_b \circ \boldsymbol{f}_b\right) - \boldsymbol{p}\right)\right) + \boldsymbol{\nabla}_{\alpha_b}\left(\beta \sum_{b=1}^{B} TV(\boldsymbol{\alpha}_b) + \gamma \sum_{b=1}^{B} ||\boldsymbol{\alpha}_b - \boldsymbol{d}_b||_2^2\right)$$

- $\beta, \gamma = 0.01$

Improved AIR

- **Improved AIR separates optimization into two steps:**

Rawdata Fidelity | Regularization

$$C(\boldsymbol{\alpha}) = ||\mathbf{X}\left(\sum_{b=1}^{B} \boldsymbol{\alpha}_b \circ \boldsymbol{f}_b\right) - \boldsymbol{p}||_{\mathbf{W}}^2 + \beta \sum_{b=1}^{B} TV(\boldsymbol{\alpha}_b) + \gamma \sum_{b=1}^{B} ||\boldsymbol{\alpha}_b - \boldsymbol{d}_b||_2^2$$

One iteration =

1. Step: Optimization of rawdata fidelity with OS-SART

⇩

SART image $\boldsymbol{\alpha}_{SART}$

2. Step: Optimization of regularization terms with gradient descent

⇩

Regularized image $\boldsymbol{\alpha}_{Reg}$

Linear Combination: $\boldsymbol{\alpha} = (1-\tau)\boldsymbol{\alpha}_{SART} + \tau\boldsymbol{\alpha}_{Reg}$

- $\beta, \gamma = 0.01$ (different weighting between the penalty terms is possible)

Improved AIR

- Improved AIR algorithm uses a method similar to the improved total variation (iTV[1]) method.
- A new condition is introduced with which a new τ is determined automatically in each iteration.

→ Condition for the rawdata fidelity after each iteration:

$$\underbrace{||\mathbf{X}\boldsymbol{f}^n - \boldsymbol{p}||_2^2}_{\text{Rawdata fidelity of the image from current iteration}} = (1-\omega)\underbrace{\epsilon_{SART}^n}_{\text{SART rawdata fidelity from current iteration}} + \omega \underbrace{\epsilon}_{\text{Rawdata fidelity: image from previous iteration}}{}^{n-1}, \qquad \omega \in [0,1]$$

Rawdata fidelity of the image from <u>current iteration</u>

SART rawdata fidelity from <u>current iteration</u>

Rawdata fidelity: image from <u>previous iteration</u>

with

$$\boldsymbol{f}^n = \sum_{b=1}^{B} \left((1-\tau)\boldsymbol{\alpha}_{b,SART} + \tau \boldsymbol{\alpha}_{b,Reg} \right) \circ \boldsymbol{f}_b$$

[1]Ritschl, Bergner, Fleischmann and Kachelrieß, "Improved total variation-based CT image reconstruction applied to clinical data", Phys. *Med. Biol. 56(6), pp. 1554-1561, 2011*

Improved AIR

- **Linear combination of the SART and the regularized image:**

$$\boldsymbol{\alpha} = (1-\tau)\boldsymbol{\alpha}_{SART} + \tau\boldsymbol{\alpha}_{Reg}, \qquad \tau \in [0,1]$$

$$\tau_{1,2} = A \pm \sqrt{A^2 + B}$$

$$A = \frac{\mathbf{X}\boldsymbol{f}^n_{SART}(\mathbf{X}\boldsymbol{f}^n_{SART} - \mathbf{X}\boldsymbol{f}^n_{Reg} - \boldsymbol{p}) + \mathbf{X}\boldsymbol{f}^n_{Reg}\boldsymbol{p}}{(\mathbf{X}\boldsymbol{f}^n_{SART} - \mathbf{X}\boldsymbol{f}^n_{Reg})^2}$$

$$B = \omega(\epsilon^{n-1} - \epsilon^n_{SART})$$

f_{SART}/f_{Reg} = AIR CT images calculated using only α_{SART}/ α_{Reg}

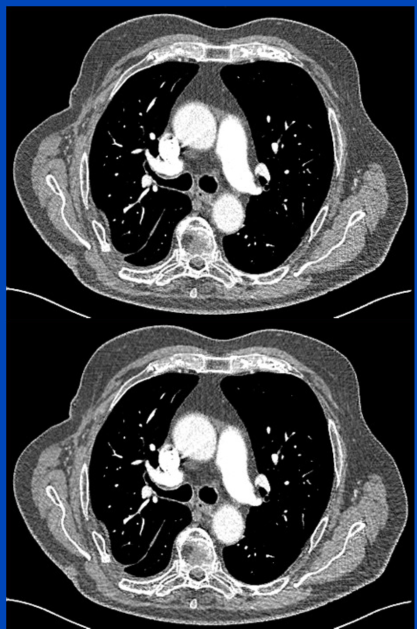
- **Parameter ω chosen manually/globally.**
  - **Sets rawdata fidelity value for each iteration**
  - **Controls the strength of the regularization**
- **ω=0.2 was choosen.**

Performance

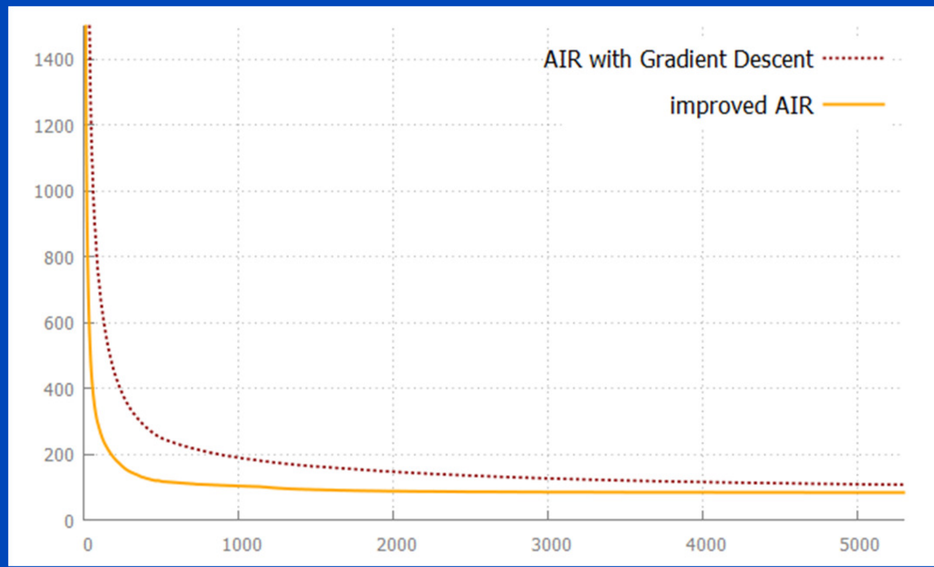
- **High quality images can be acquired after a couple 1000 iterations of the gradient descent imple-mentation and after 200-300 iterations of the improved algorithm.**

AIR (Gradient Descent)

Improved AIR



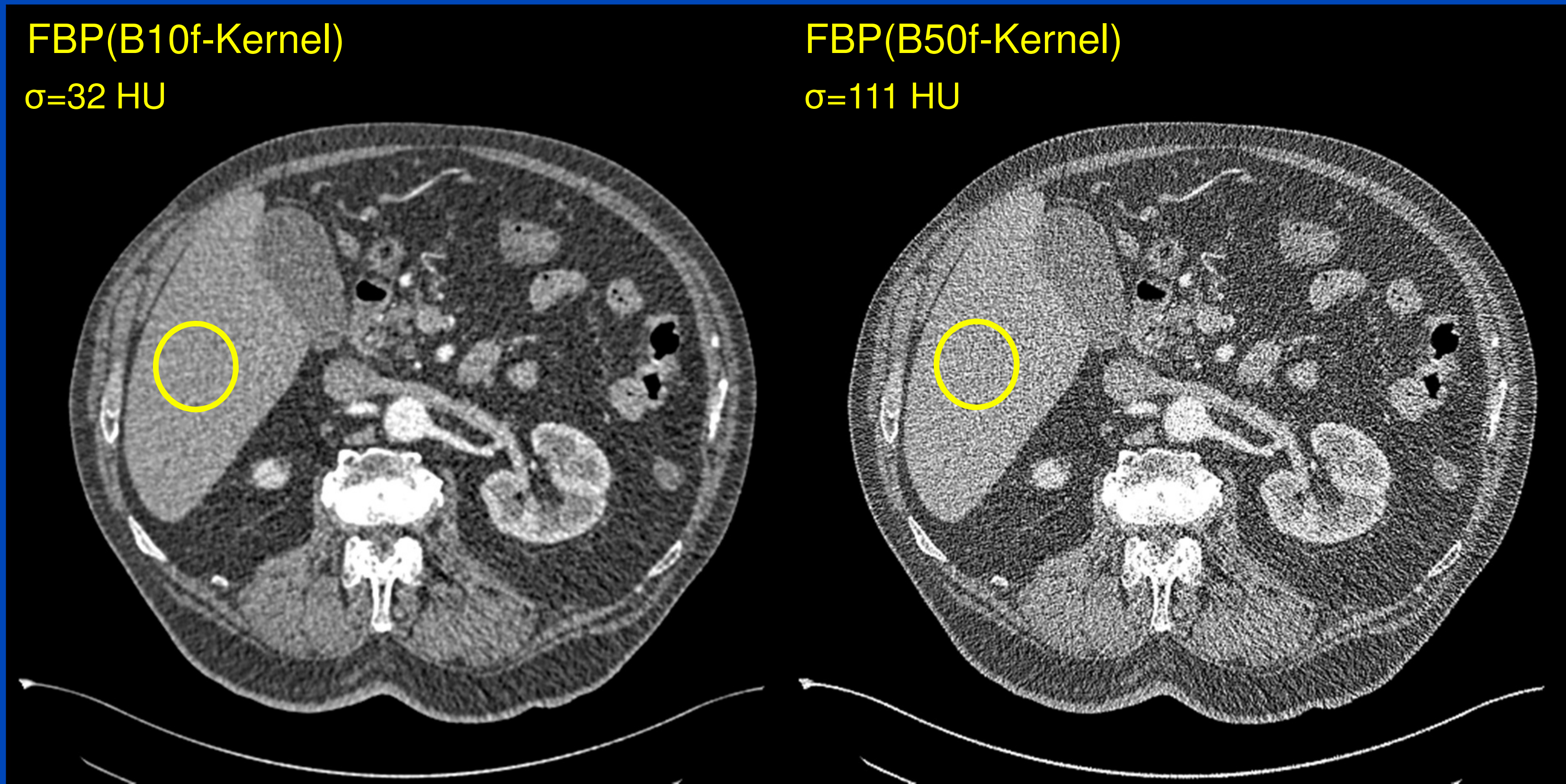
Rawdata Fidelity



Computation Time /s

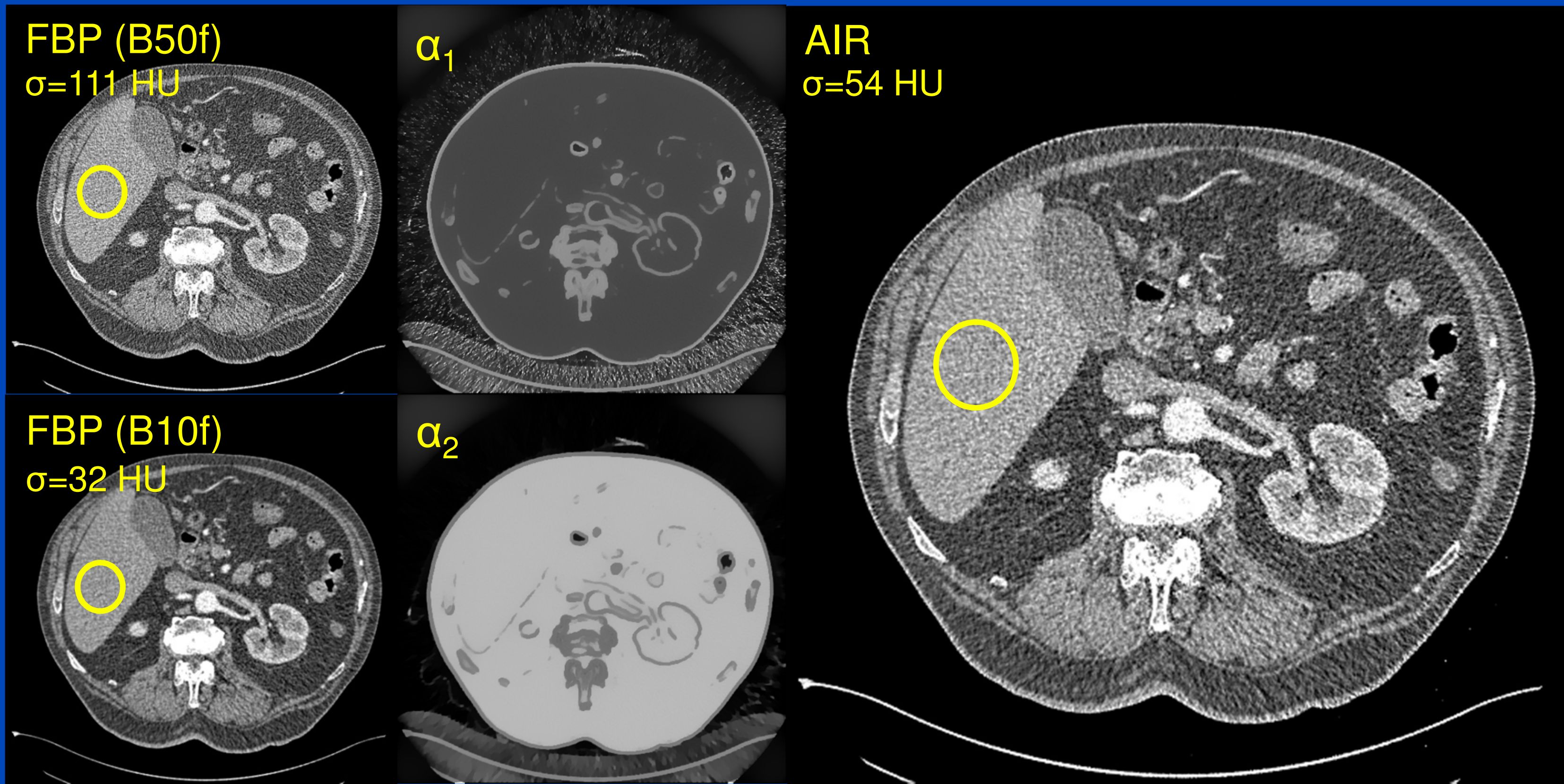
Convergence plots of the rawdata fidelity

AIR Applied to Clinical Data



C/W = 0/500 HU   C/W = 0/500 HU

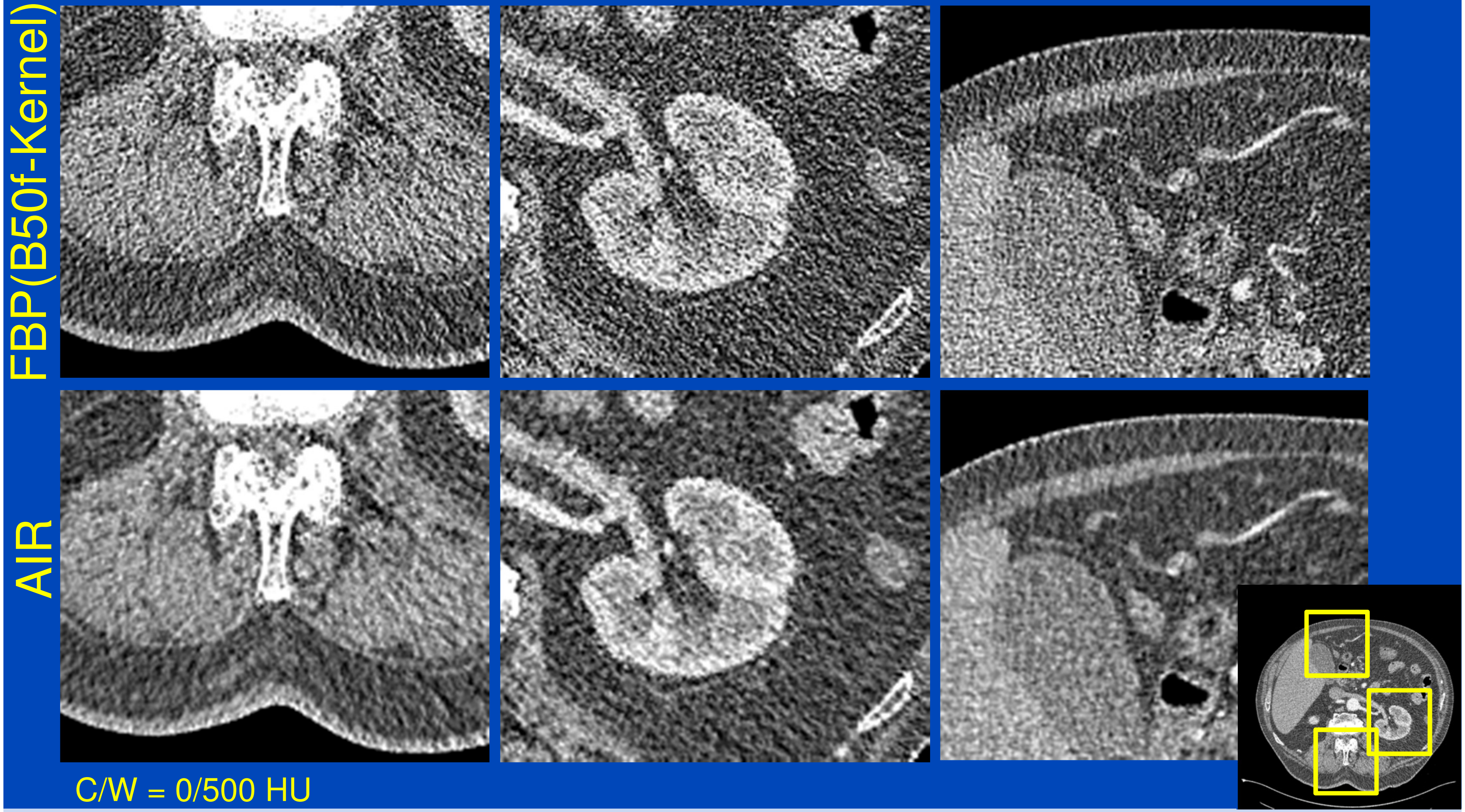
AIR Applied to Clinical Data



C/W = 0/500 HU　　C/W = 0.5/1　　C/W = 0/500 HU

ω=0.2

AIR Applied to Clinical Data

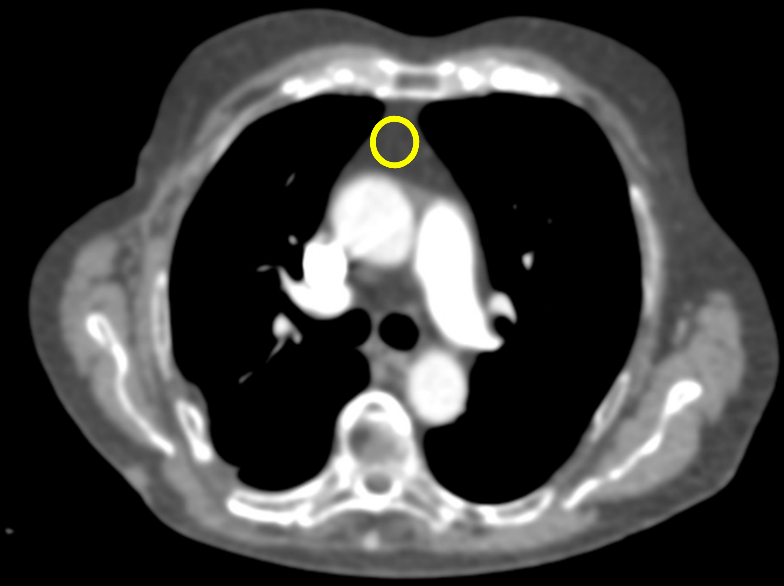


C/W = 0/500 HU

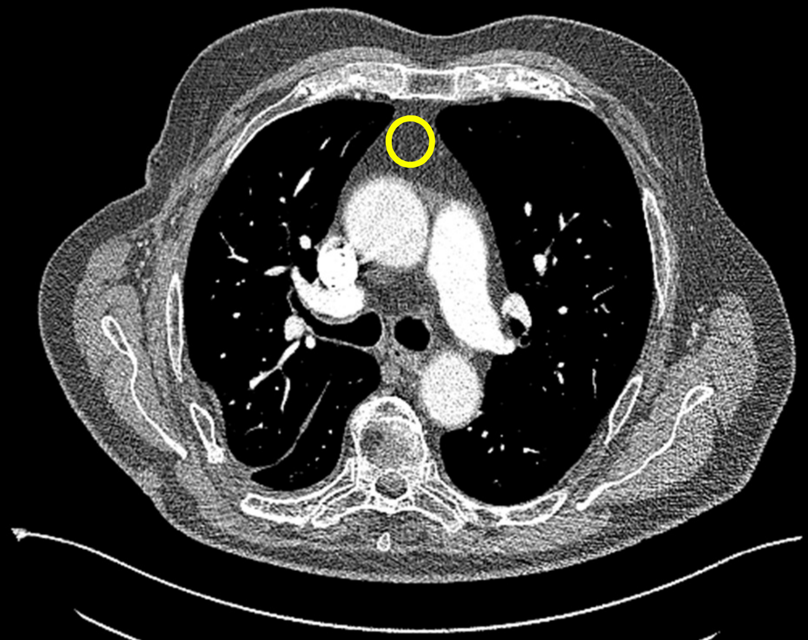
ω=0.2

AIR Applied to Clinical Data

Smooth FBP
(Filtered with a Gaussian filter)
σ=6 HU



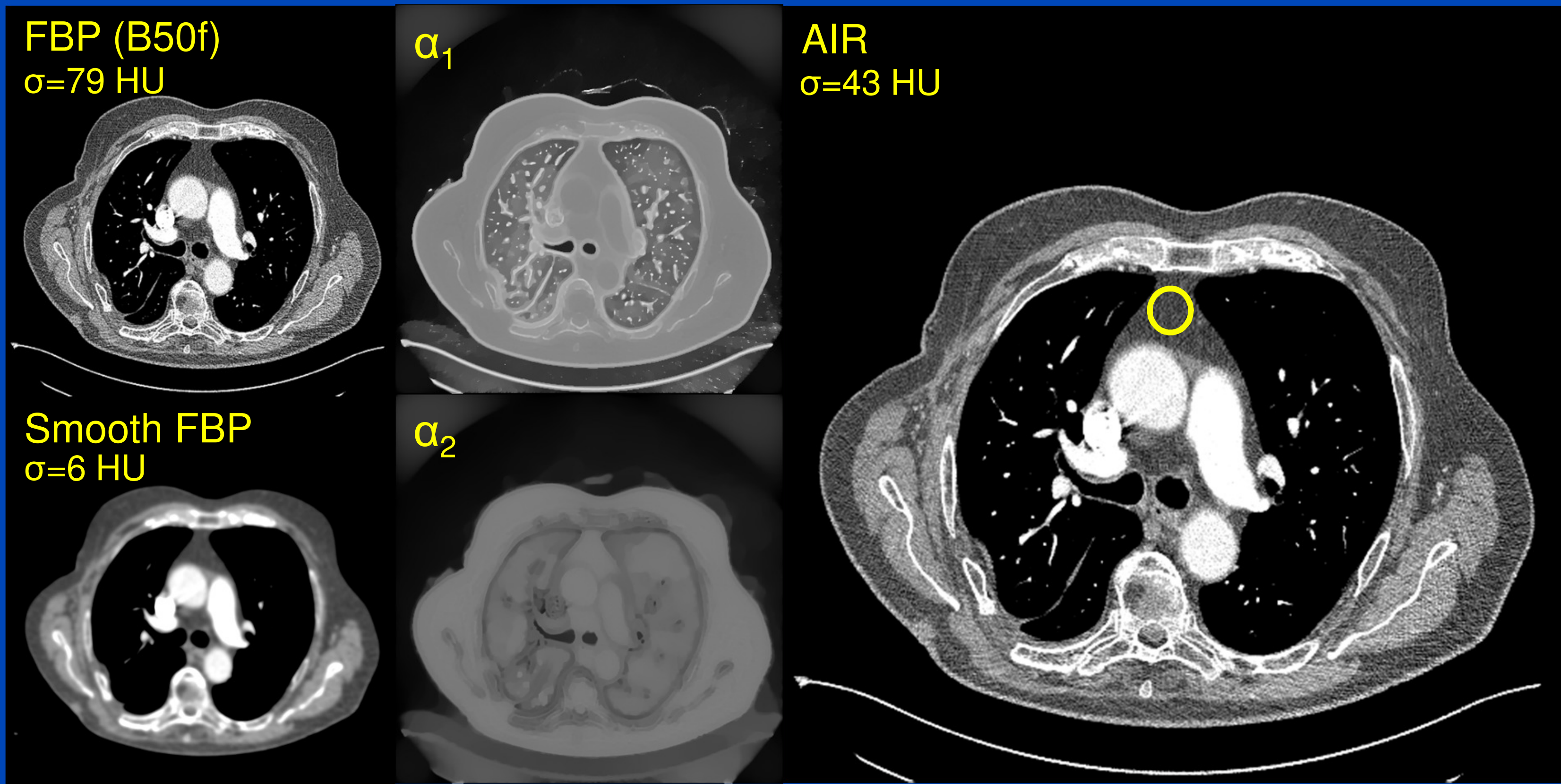
FBP(B50f-Kernel)
σ=79 HU



C/W = 0/500 HU C/W = 0.5/1 C/W = 0/500 HU

Noise levels in soft tissue (yellow circle): FBP(B50f): σ=79 HU, smooth FBP: σ=6 HU

AIR Applied to Clinical Data

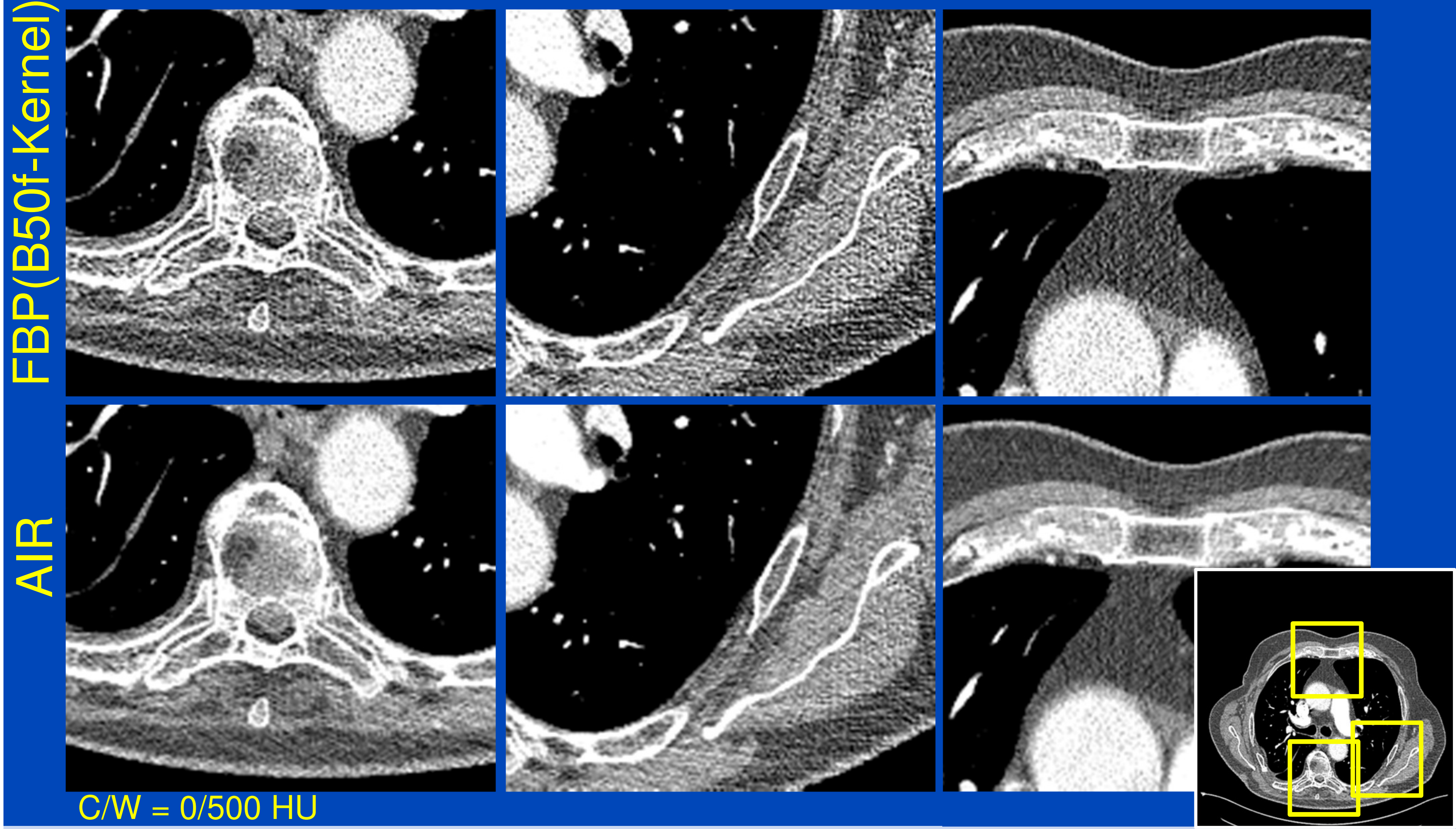


C/W = 0/500 HU C/W = 0.5/1 C/W = 0/500 HU

Noise levels in soft tissue (yellow circle): FBP(B50f): σ=79 HU, smooth FBP: σ=6 HU
AIR: σ=43 HU

ω=0.2

AIR Applied to Clinical Data



ω=0.2

Modulation Transfer Function

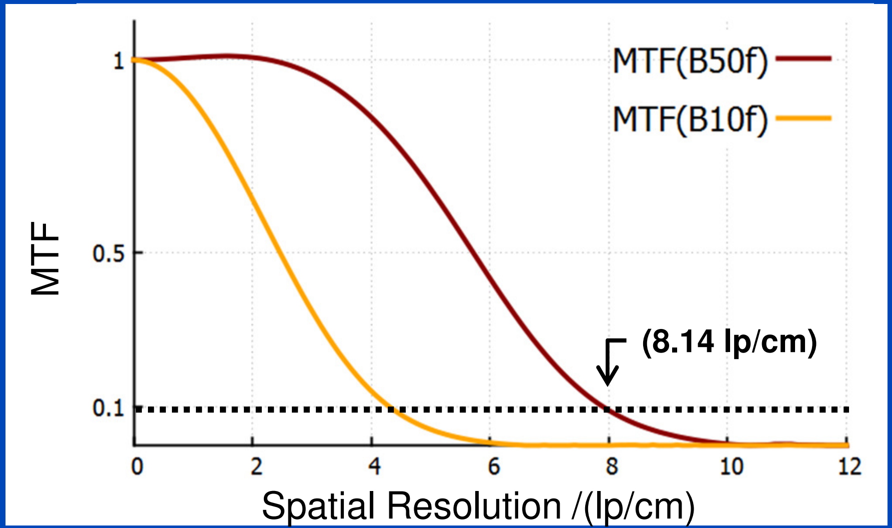
- If an MTF or another image quality metric is defined for the basis images it can be estimated for every voxel of the AIR image.
- MTF of a B50f/B10f-Kernel was measured at a Definition Flash Scanner.

$$\mathrm{MTF}(j,\rho) = \sum_{n}^{B} \alpha_b^j \, \mathrm{MTF}_b(\rho)$$

α_b^j = voxel j of weighting image b

B = number of basis images

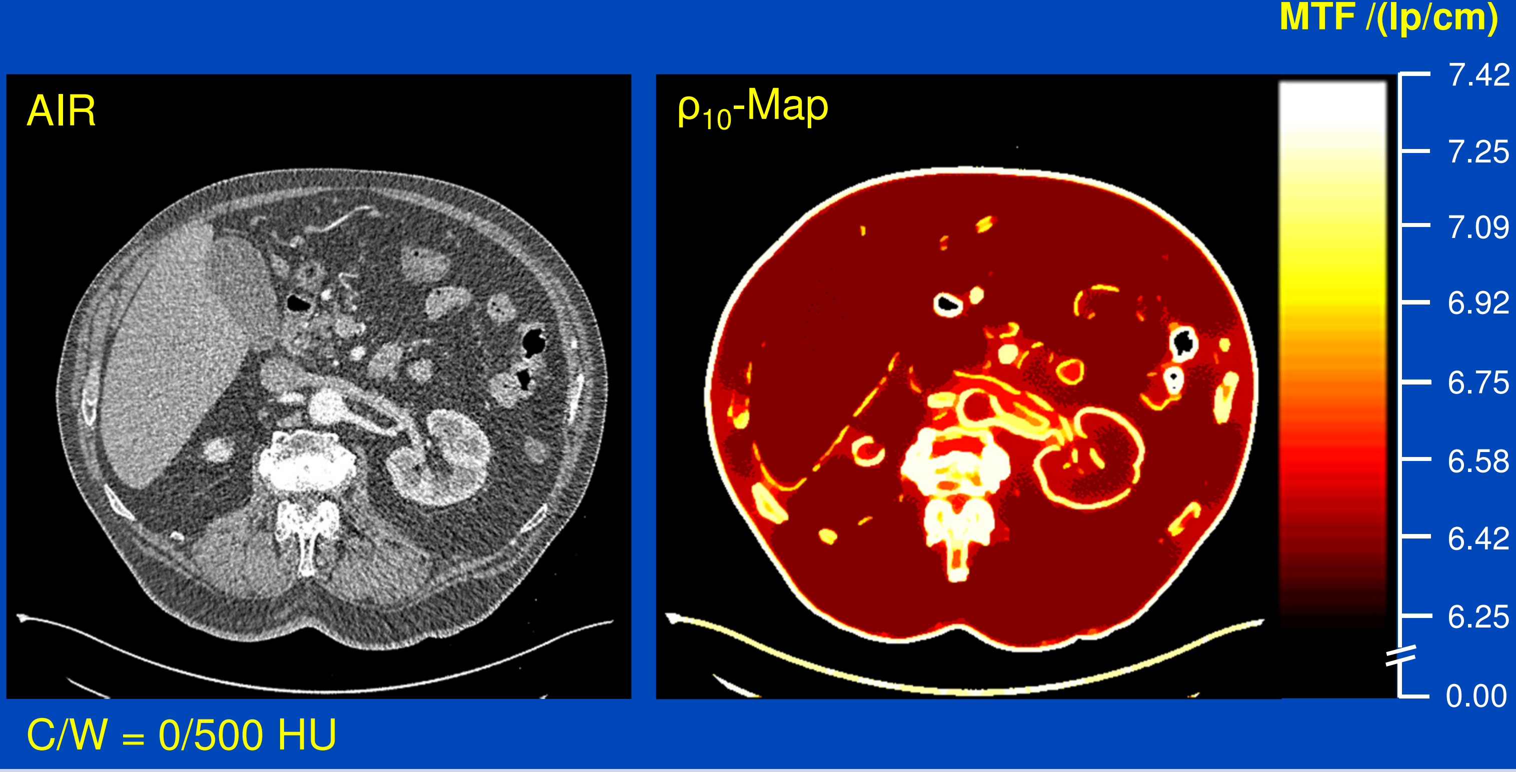
MTF_b = MTF of the basis image b



Modulation Transfer Functions for the Basis Images

Modulation Transfer Function

- The MTF is computed for each voxel.
- The “10%-value“ of the MTF ρ_{10} for each voxel is displayed as a map.



Conclusion

- Optimized AIR algorithm improves performance by a factor of about 5-10.
- Noise can be significantly reduced while spatial resolution at edges is mostly maintained.
- Predictions for image quality metrics based on the basis images are possible.

Thank You!

This presentation will soon be available at www.dkfz.de/ct.

Parts of the reconstruction software RayConStruct IR were provided by RayConStruct® GmbH, Nürnberg, Germany.