Exact Analytical Image Reconstruction for Patient-Specific Arbitrary 180°-Complete CT Scan Trajectories

Michael Knaup[1], Jan Kuntz[1], Ludwig Ritschl[2], Phil Steininger[3], and Marc Kachelrieß[1]

[1]German Cancer Research Center (DKFZ), Heidelberg, Germany
[2]Ziehm Imaging GmbH, Nürnberg, Germany
[3]medPhoton GmbH, Salzburg, Austria

Part 1:

INTRODUCTION: ARBITRARY SCAN TRAJECTORIES

Introduction

Several modern CT systems allow scan trajectories beyond the standard circle trajectories.

C-arm Rotate+Shift scan:

Vision RFD 3D mobile C-arm system, Ziehm Imaging GmbH, Nürnberg, Germany

Introduction

Several modern CT systems allow scan trajectories beyond the standard circle trajectories.

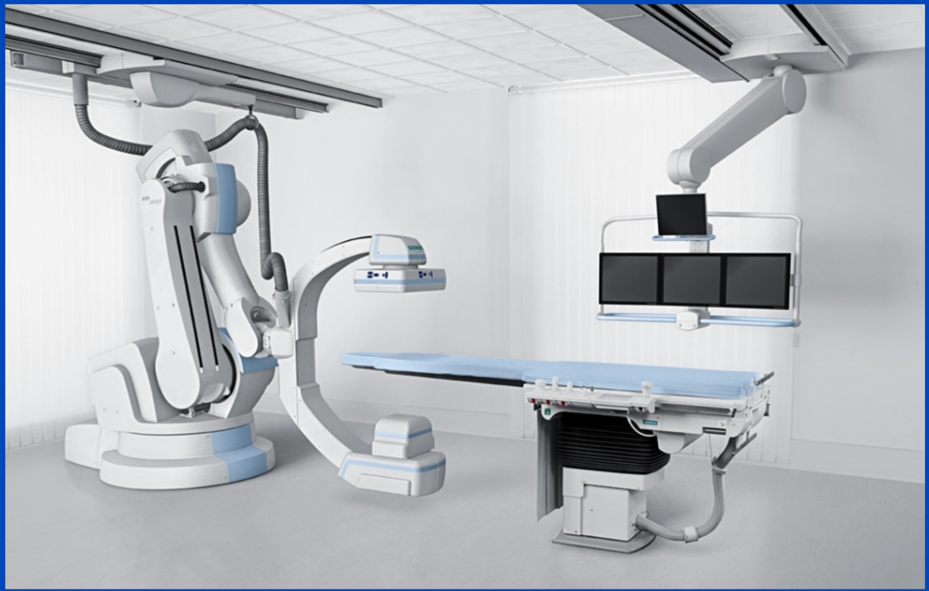
Independent source and detector movement with virtual isocenter:

Patient Alignment Imaging Ring (PAIR), medPhoton GmbH, Salzburg, Austria

Introduction

Several modern CT systems allow scan trajectories beyond the standard circle trajectories.

Other examples:



Artis Zeego
Siemens Healthcare, Forchheim, Germany

TrueBeam
Varian Medical Systems, Palo Alto, CA, USA

Purpose

- To allow for an analytical image reconstruction, the rawdata must be pre-weighted to account for the 180° redundencies of the measured rays.
- A dedicated pre-weight must be derived for each individual scan trajectory (e.g. Parker weight [1] for a circular short scan).
- Therefore, the implementation of an arbitrary scan trajectory is very time-consuming and, e.g., patient specific trajectories [2] cannot be easily realized.
- Consequently, as of today, there is no commercial system realizing arbitrary trajectories.

Aim

- To develop a general weighting scheme which can be used for pre-weighting of rawdata from any arbitrary scan trajectory.
- In this work we restrict ourselves to in-plane trajectories which provide 180° complete rawdata.

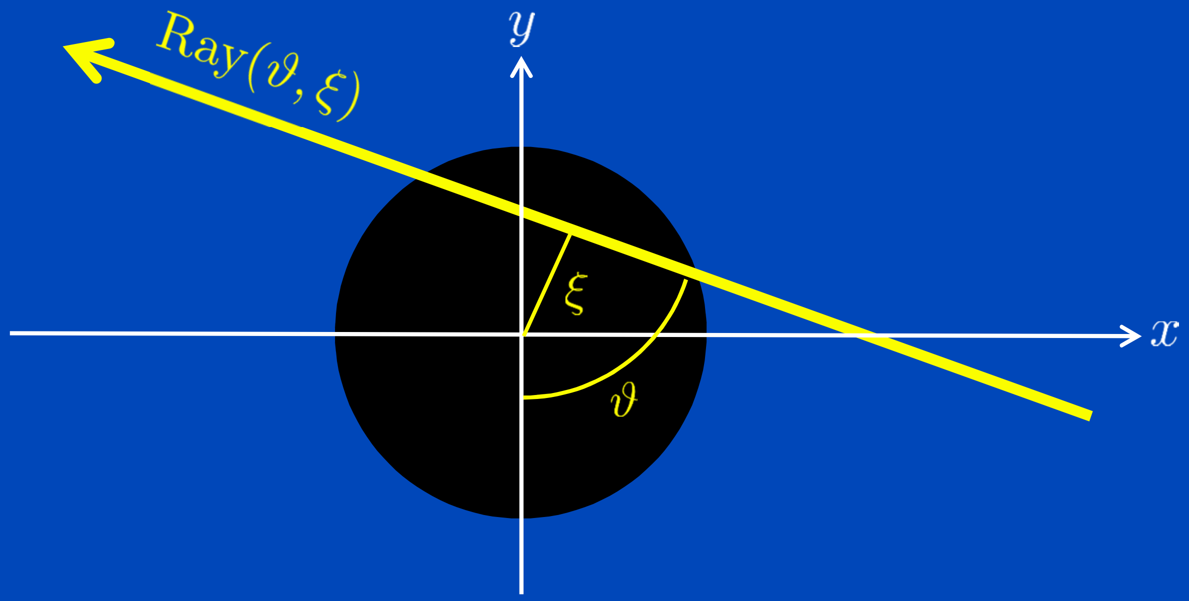
Part 2:

GENERAL WEIGHTING SCHEME

Virtual Parallel Geometry

- **A ray in parallel geometry is defined by the parameters (ϑ, ξ). It runs through all points (x, y) with**

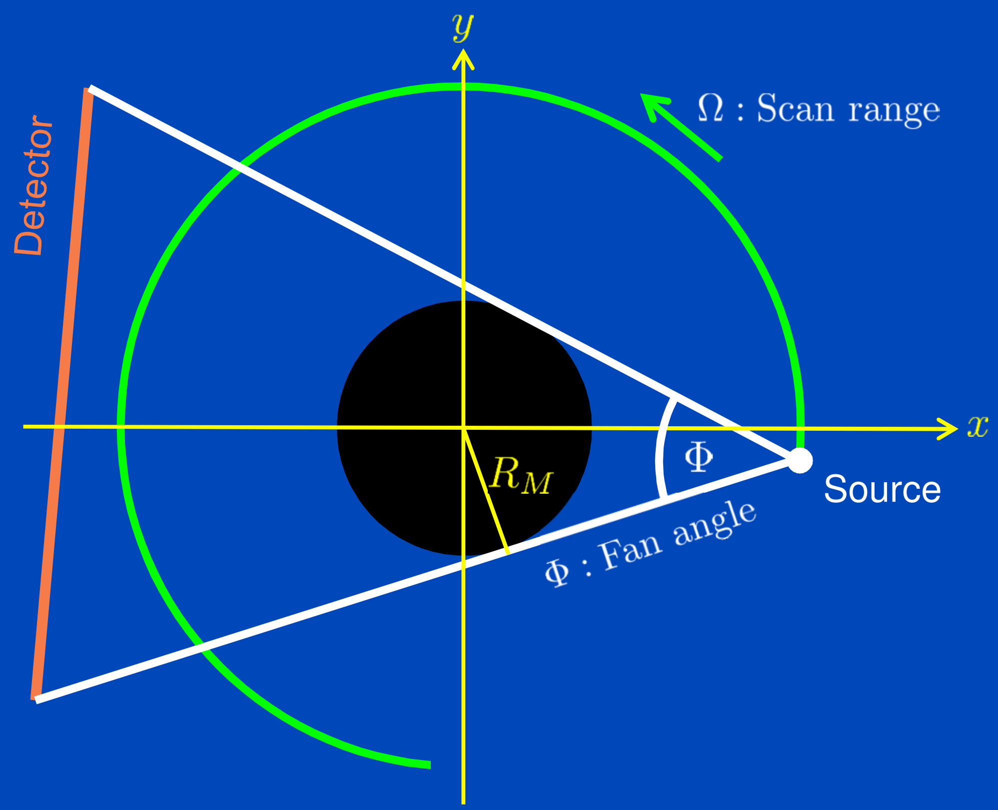
$$x \cos\vartheta + y \sin\vartheta = \xi$$

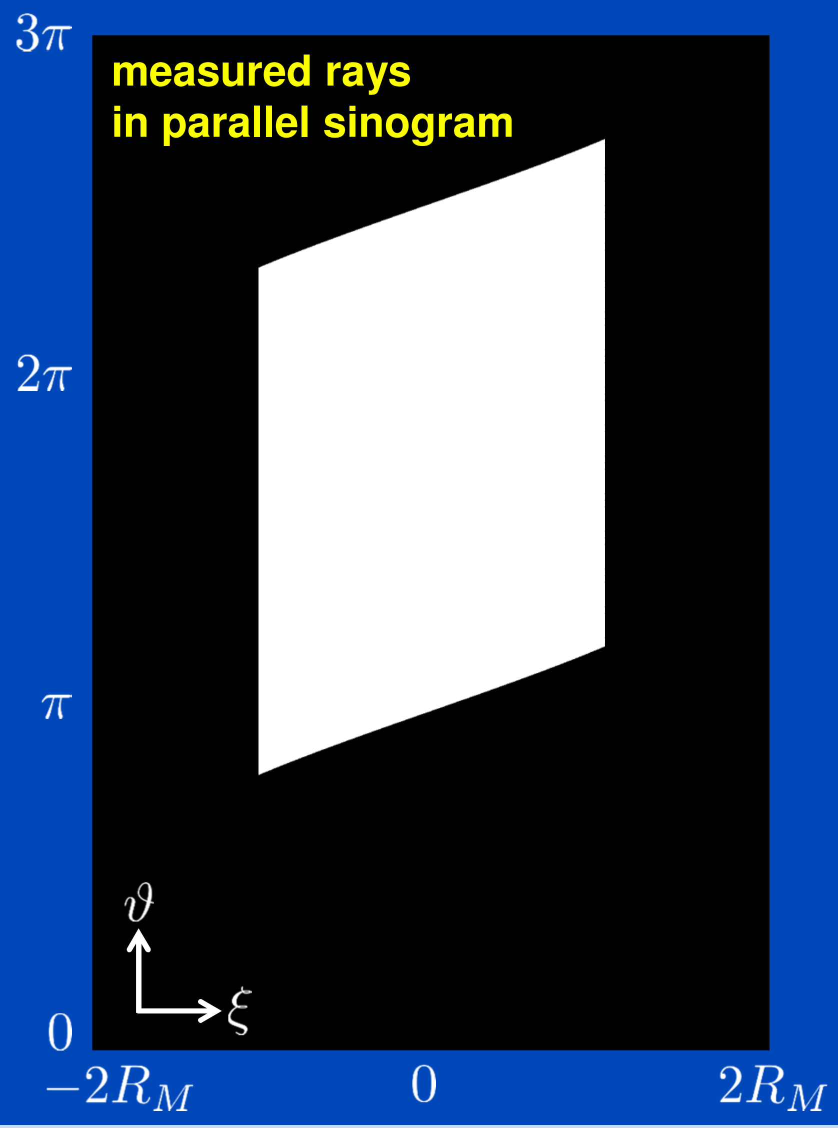


- **Identical rays:** $\left(\vartheta + h\pi, (-1)^h \xi\right) \quad \forall h \in \mathbb{Z}$

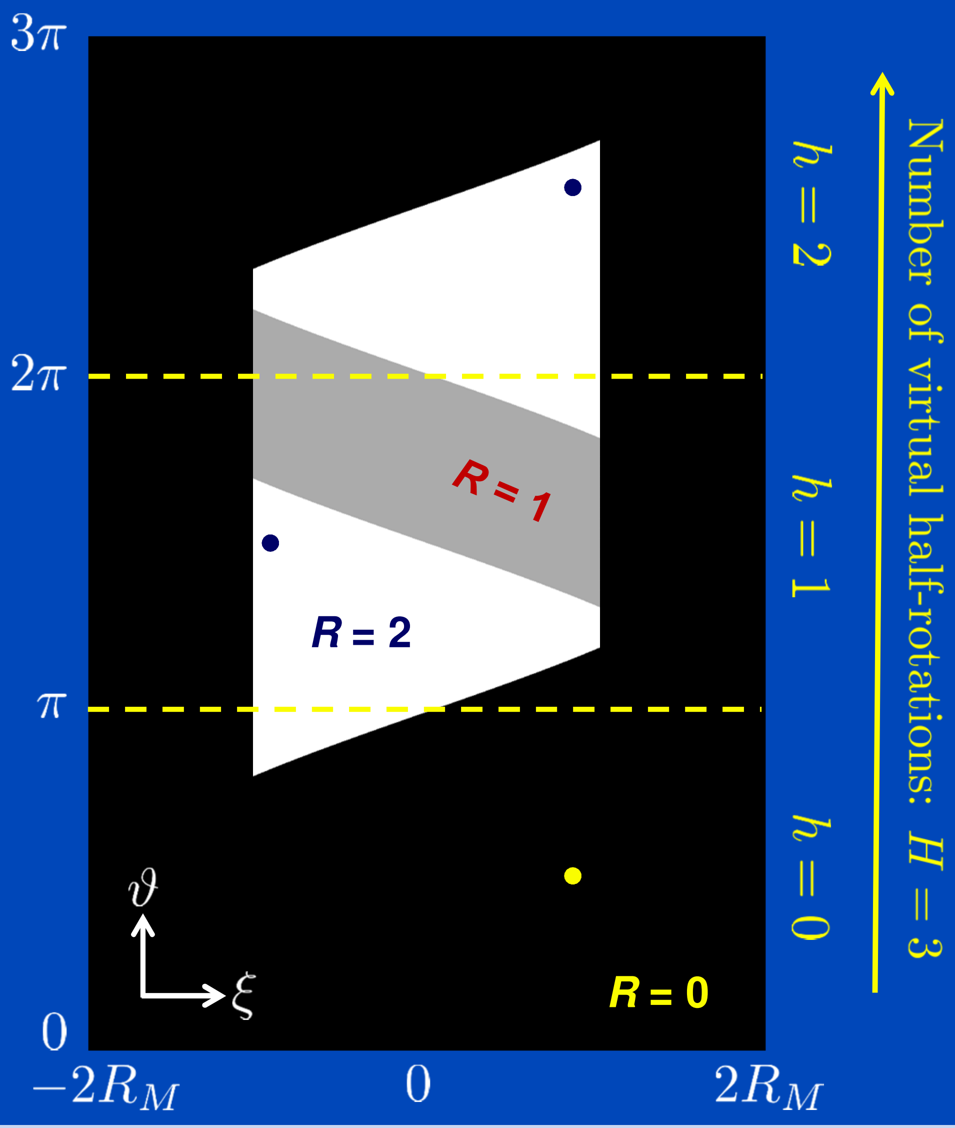
Short Scan

In the following, we consider a short scan, i.e. $\Omega < 360°$.



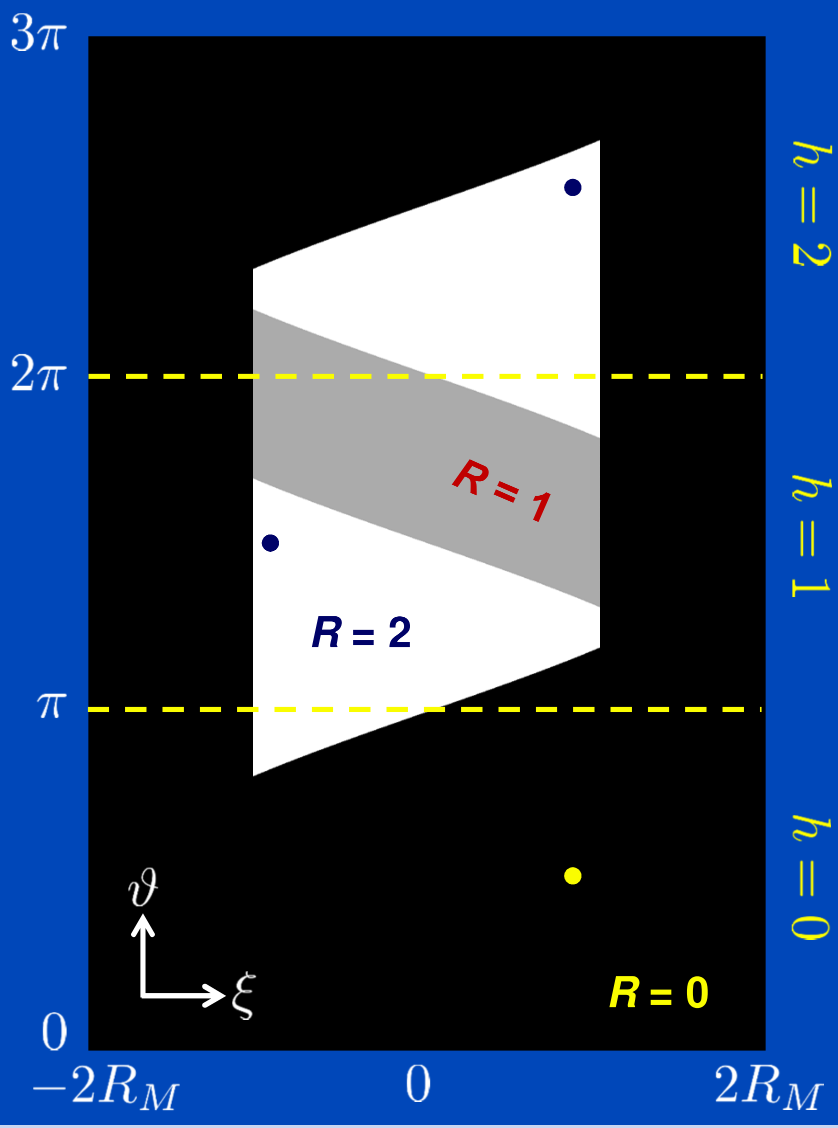


Step 1: Counting Redundancies



- **For each ray (ϑ, ξ) we count the number of fan projections covering this ray.**
- **This number of fan projections is called the redundancy $R(\vartheta, \xi)$ of this ray.**
- **The list of fan projections is mapped to a list of virtual half rotations h such that adjacent rays in the extended parallel sinogram are covered by adjacent fan projections.**

Step 2: Weights in Parallel Geometry

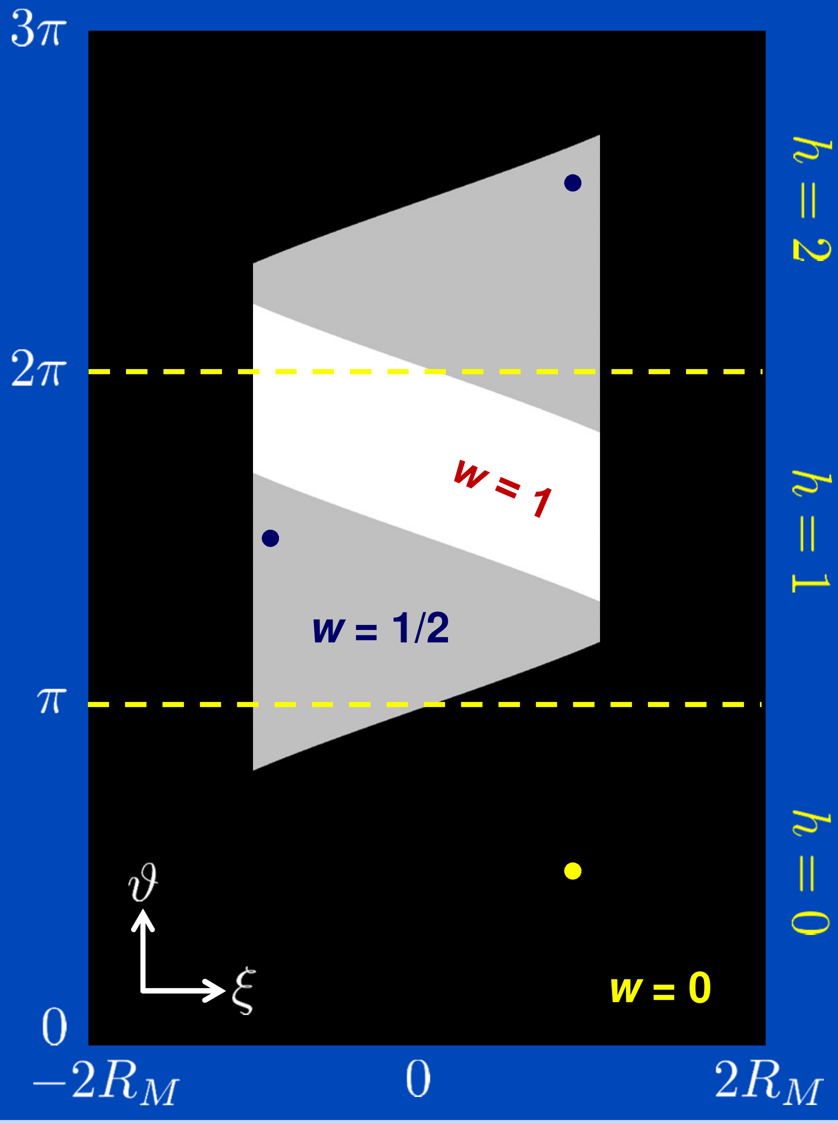


- Now we look for appropriate weights $0 \leq w(\vartheta, \xi) \leq 1.$

- To account correctly for the redundancies, the weights must fulfill the constraint

$$\sum_{h=0}^{H-1} w\left(\vartheta + h\pi, (-1)^h \xi\right) = 1 \quad \forall \vartheta, \xi$$

Step 2: Weights in Parallel Geometry



- **Now we look for appropriate weights** $0 \leq w(\vartheta,\xi) \leq 1$.

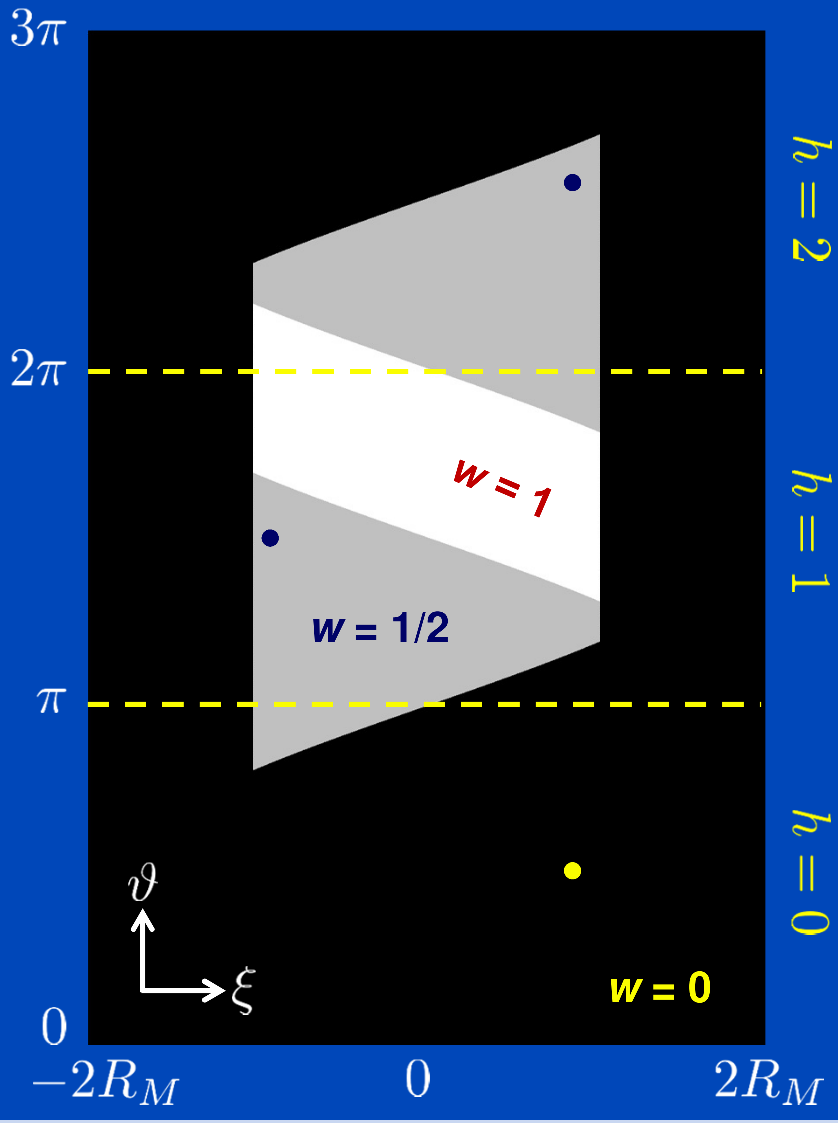
- **To account correctly for the redundancies, the weights must fulfill the constraint**

$$\sum_{h=0}^{H-1} w\left(\vartheta + h\pi, (-1)^h\,\xi\right) = 1 \quad \forall \vartheta, \xi$$

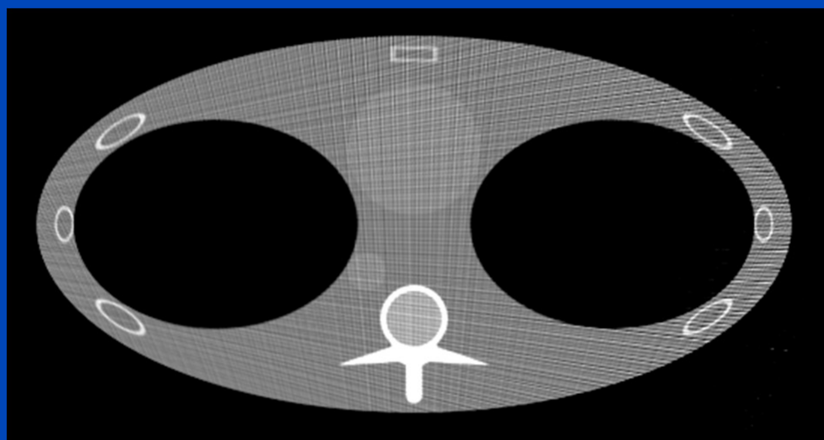
- **An obvious choice would be**

$$w\left(\vartheta + h\pi, (-1)^h\,\xi\right) = $$
$$= \begin{cases} R(\vartheta,\xi)^{-1} & \text{if ray is measured.} \\ 0 & \text{if ray is not measured.} \end{cases}$$

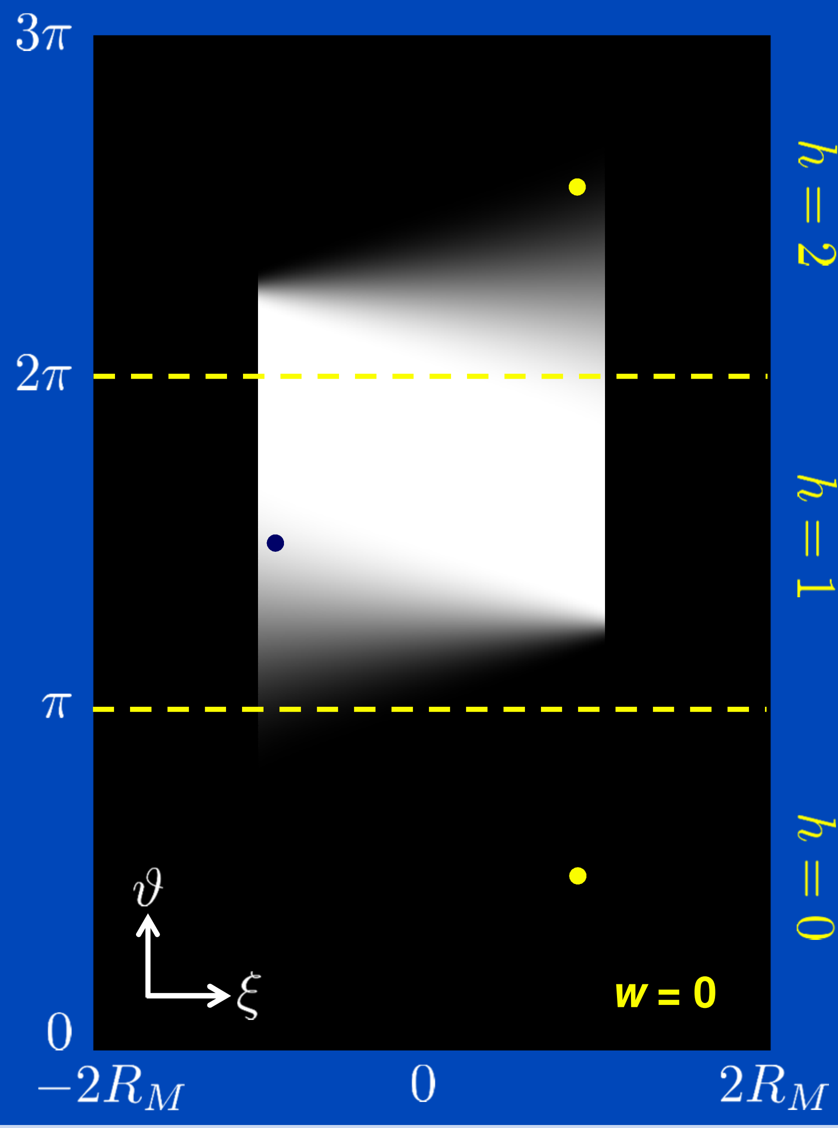
Step 2: Weights in Parallel Geometry



- However, this obvious choice results in non-continuous weights which produce unwanted streak artifacts in the final image due to the discreteness of the sampling:

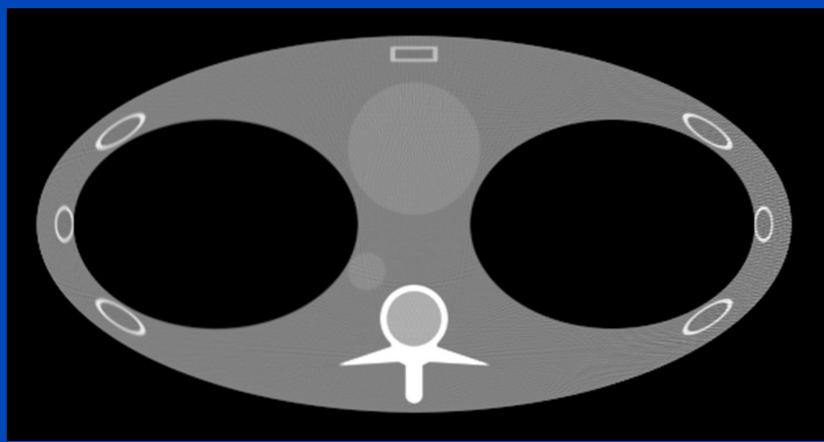


Step 2: Weights in Parallel Geometry



- **Therefore, we must smooth the weights under the constraint**

$$\sum_{h=0}^{H-1} w\left(\vartheta + h\pi, (-1)^h\,\xi\right) = 1 \quad \forall \vartheta, \xi$$



Step 3: Smoothing Weights

We smooth the weights by minimizing the following cost function:

$$C = \sum_{\vartheta,\xi} \left[A_{\vartheta,\xi} + \beta B_{\vartheta,\xi} + \lambda_{\vartheta,\xi} \left(1 - \sum_{h=0}^{H-1} w\big(\vartheta + h\pi, (-1)^h\,\xi\big) \right) \right]$$

with

$$A_{\vartheta,\xi} = \sum_{h=0}^{H-1} \left[\left(\frac{\partial}{\partial\vartheta} w\big(\vartheta + h\pi, (-1)^h\,\xi\big) \right)^2 + \left(\frac{\partial}{\partial\xi} w\big(\vartheta + h\pi, (-1)^h\,\xi\big) \right)^2 \right]$$

(minimizes 1st derivatives)

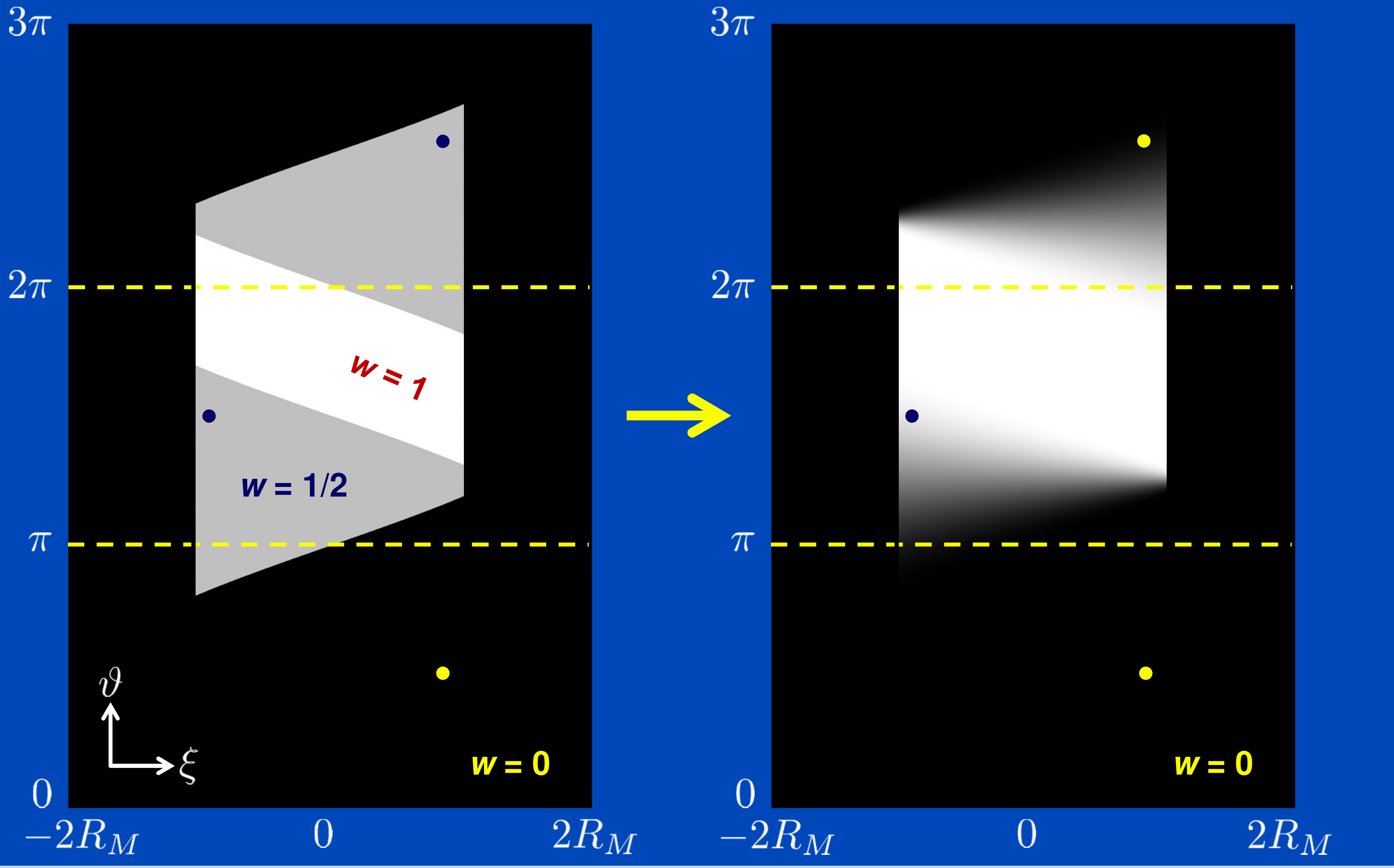
$$B_{\vartheta,\xi} = \sum_{h=0}^{H-1} \left[\left(\frac{\partial^2}{\partial\vartheta^2} w\big(\vartheta + h\pi, (-1)^h\,\xi\big) \right)^2 + \left(\frac{\partial^2}{\partial\xi^2} w\big(\vartheta + h\pi, (-1)^h\,\xi\big) \right)^2 \right]$$

(minimizes 2nd derivatives)

The cost function will be minimized by a gradient descent approach with respect to the following variables:

- **All weights** $w(\vartheta, \xi)$ **with redundancy** $R(\vartheta, \xi) > 1.$
- **The Lagrange multipliers** $\lambda_{\vartheta,\xi}$ **which enforce the constraint.**

Step 3: Smoothing Weights



Part 3:

SIMULATION STUDIES

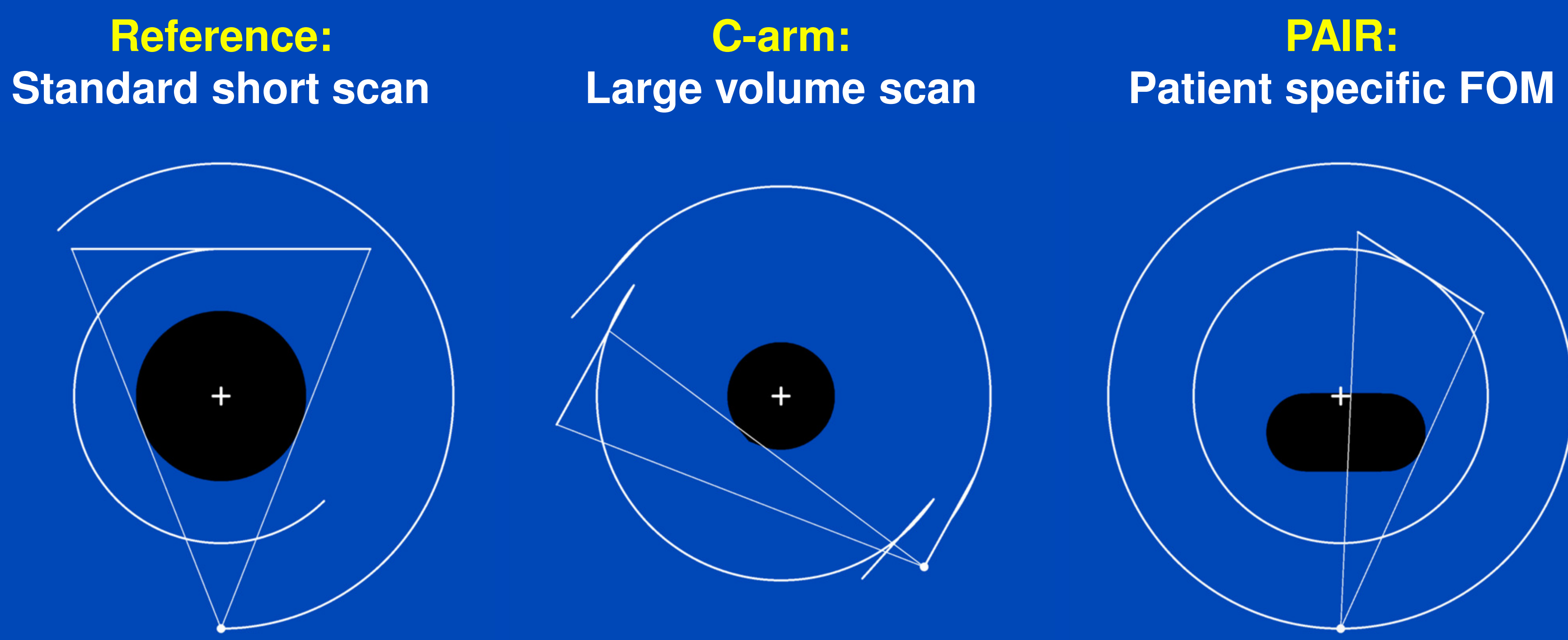
Simulation Studies

- We simulated example trajectories of a Ziehm Vision RFD 3D mobile C-arm system and a Patient Alignment Imaging Ring (PAIR) system.
  - A C-arm large volume scan [3] which is realized as two rotate+shift scans [4] with a virtual shifted detector.
  - A PAIR scan which realizes a patient-specific field of measurement.
- The trajectories are not yet actually implemented in the respective devices but demonstrate the potentials of these systems.
- As a reference, we simulated a standard short scan in the PAIR geometry with an artificial large detector.

Simulation Studies

- X-ray photon noise was added to the simulated rawdata. All scans were simulated at the same total patient dose.
- The rawdata were pre-weighted using the proposed general weighting scheme.
- Finally, the pre-weighted rawdata were reconstructed using a standard Feldkamp-Davis-Kress (FDK) algorithm [5].

Simulated Geometries



Field of measurement (FOM)

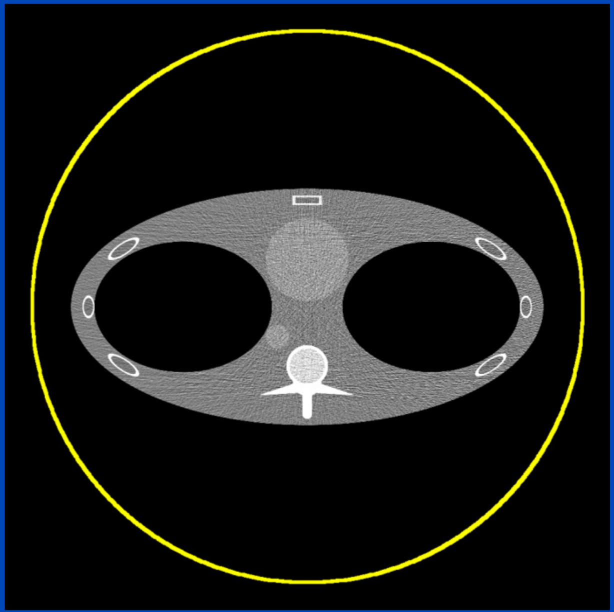
\+ Virtual (C-arm) or physical (PAIR) isocenter, resp.

General Weights

Reference: Standard short scan | **C-arm:** Large volume scan | **PAIR:** Patient specific FOM

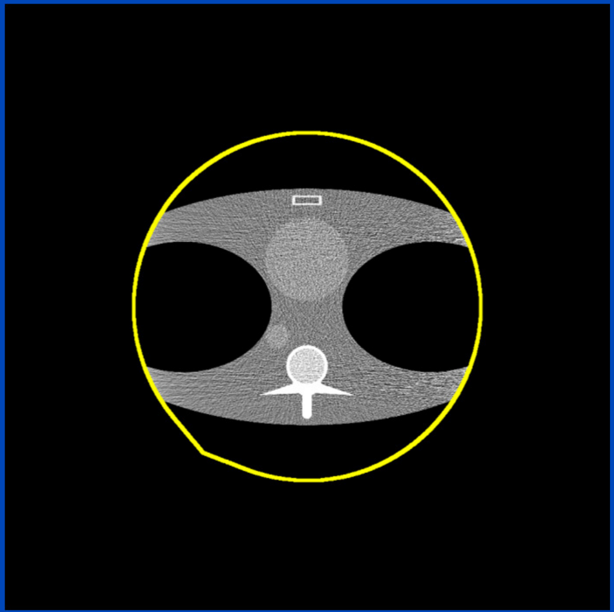
Reconstructions

Reference:
Standard short scan

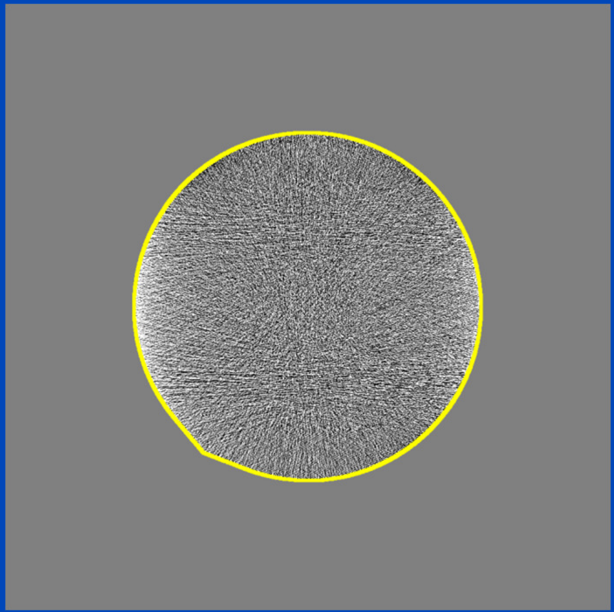


(C/W) = (0 HU / 500 HU)

C-arm:
Large volume scan



Difference:
C-arm minus reference

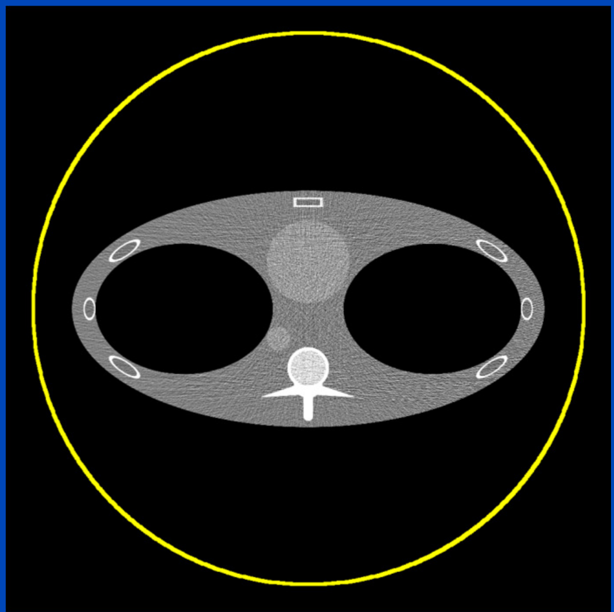


(C/W) = (0 HU / 100 HU)

Remaining differences are due to the need to detruncate the C-arm data.

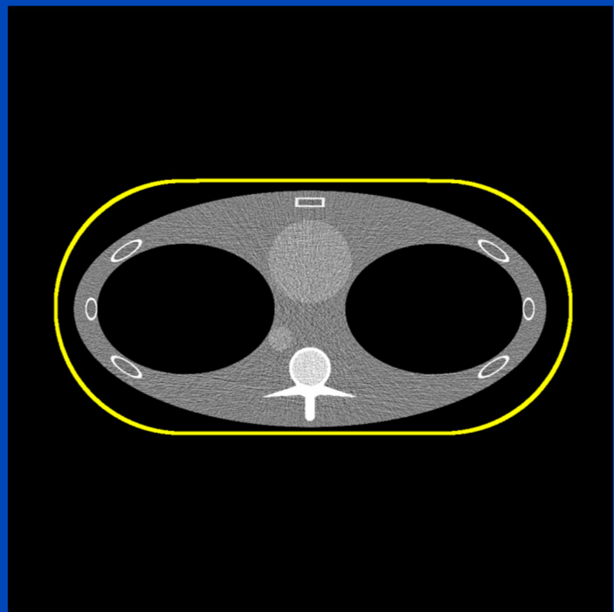
Reconstructions

Reference:
Standard short scan

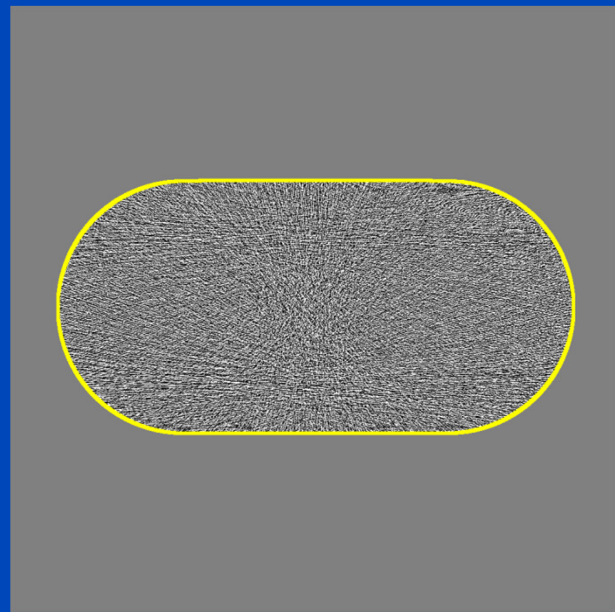


(C/W) = (0 HU / 500 HU)

PAIR:
Patient specific FOM



Difference:
PAIR minus reference



(C/W) = (0 HU / 100 HU)

Part 4:

RECONSTRUCTIONS FROM MEASURED DATA

Reconstructions from Measured data

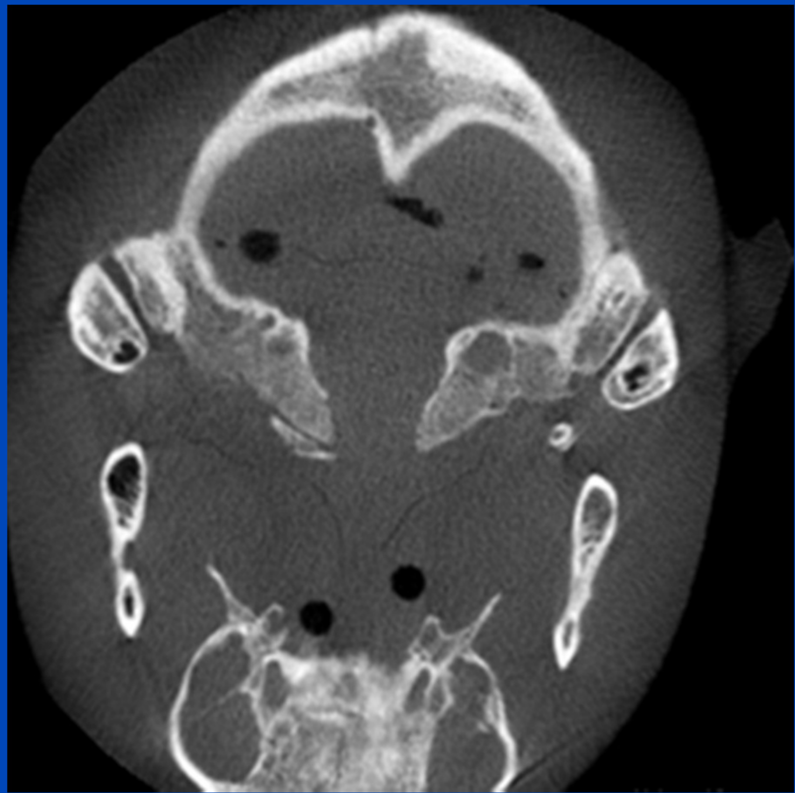
- To validate our approach, we reconstructed actually measured data from a Ziehm Vision RFD 3D mobile C-arm system and a Patient Alignment Imaging Ring (PAIR) system.
  - A C-arm rotate+shift scan [4] which allows for 180° complete data despite of a limited rotational scan range of 165° (see slide 3).
  - A PAIR scan with a virtual isocenter (see slide 4).

Ziehm C-arm System

165° scan without shift

Limited angle artifacts

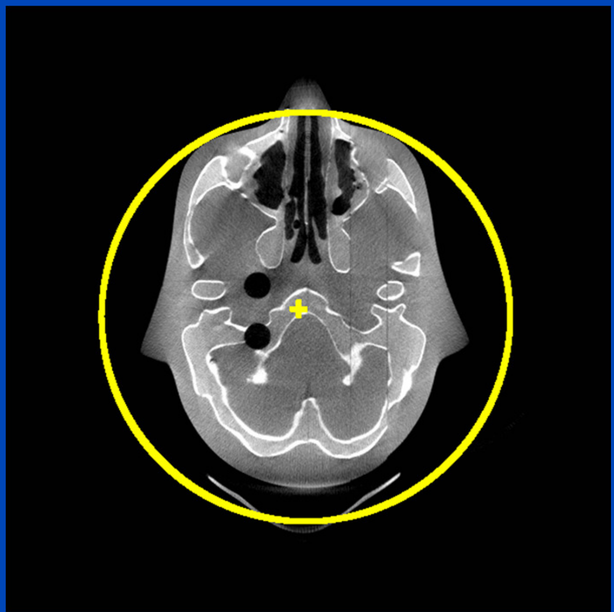
Rotate+shift scan [4]



180° complete scan

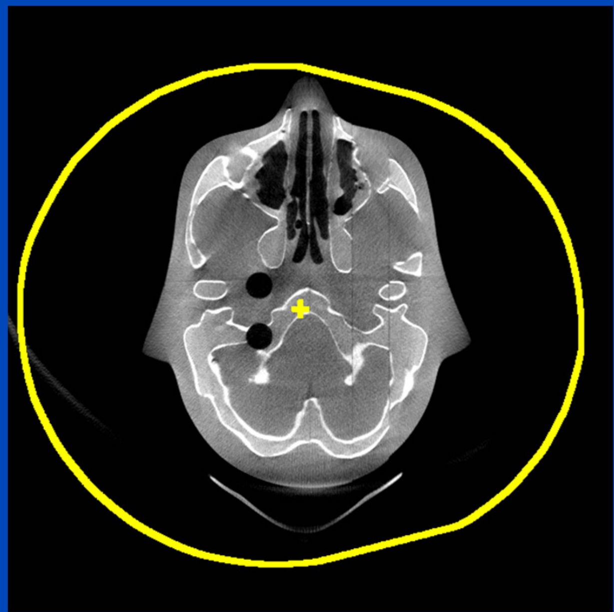
PAIR System

Standard reconstruction

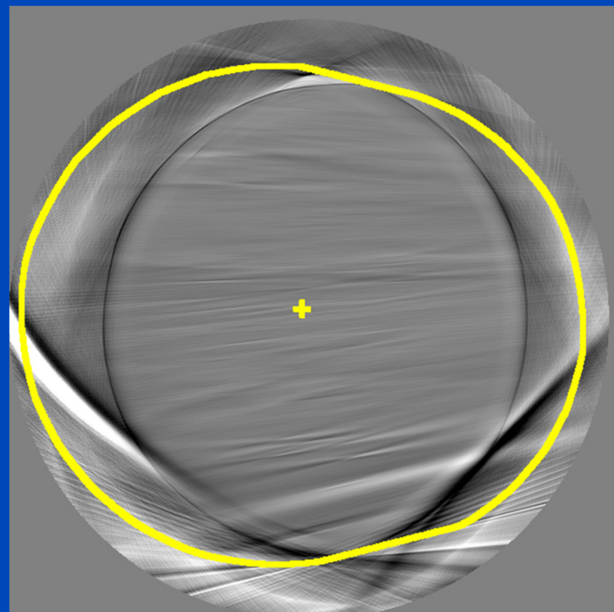


(C/W) = (0 HU / 1000 HU)

Generalized weights reconstruction with increased FOM



Difference: Generalized weights minus standard



(C/W) = (0 HU / 250 HU)

Conclusions

- We developed a general weighting scheme for arbitrary in-plane 180° complete scan trajectories.
- The general weights correctly account for 180° redundencies of all measured rays and allow for a standard analytical Feldkamp-Davis-Kress (FDK) reconstruction [5] of the pre-weighted rawdata.
- This significantly eases the implementation of new scan trajectories and allows for run-time, e.g. patient-specific trajectories [2].

References

[1] D. L. Parker, "Optimal short scan convolution reconstruction for fanbeam CT," Med. Phys., vol. 9, no. 2, pp. 254-257, 1982.

[2] J. W. Stayman, G. J. Gang, and J. H. Siewerdsen, "Task-Based Optimization of Source-Detector Orbits in Interventional Cone-beam CT," Proceedings of the 13th International Meeting on Fully Three-Dimensional Image Reconstruction in Radiology and Nuclear Medicine, pp. 55-58, 2015.

[3] J. Kuntz, C. Fleischmann, L. Ritschl, M. Knaup, and M. Kachelrieß, "Large Volume Scan Trajectory for Mobile C-Arm CBCT Systems: A Simulation Study," Poster PH265-SD-WEA4, RSNA 2015.

[4] L. Ritschl, J. Kuntz, and M. Kachelrieß, "The rotate-plus-shift C-arm trajectory part I+II," Med. Phys., submitted, 2015.

[5] L. Feldkamp, L. Davis, and J. Kress, "Practical cone-beam algorithm," Journal of the Optical Society of America, vol. 1, no. 6, pp. 612-619, 1984.

Thank You!

This presentation will soon be available at www.dkfz.de/ct.

The study was supported by the Deutsche Forschungsgemeinschaft (DFG) under grant No. KA-1678/11-1.

Parts of the reconstruction software RayConStruct-IR were provided by RayConStruct® GmbH, Nürnberg, Germany.