Basics of Tomography 3: Data Sparsity

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DEUTSCHES KREBSFORSCHUNGSZENTRUM IN DER HELMHOLTZ-GEMEINSCHAFT

What is Data Sparsity?

- No unique definition
- Less data than theoretically necessary for an exact reconstruction of a signal (Nyquist criterion)
- Sometimes it means only the special case of missing data with regularly missing data (few views)
  - Sparse data = only view-few (Sidky E.Y., Kao C.-M., Pan, X.: “Accurate image reconstruction from few-views and limited-angle data in divergent-beam CT”)
- Sometimes it includes all cases of missing data no matter where data are missing
  - Few-view and limited angle data (LaRoque S.J., Sidky E.Y., Pan X.: “Accurate image reconstruction from few-view and limited-angle data in diffraction tomography“)
  - Bunched views, missing data at the detector (Abbas et al.: “Effects of sparse sampling schemes on image quality in low-dose CT”)

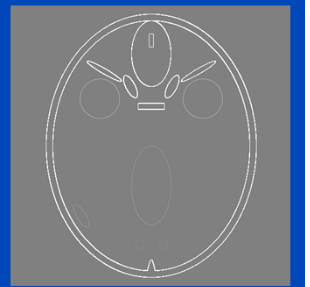
Why Data Sparsity?

- Not enough time to collect all data (e.g. in MRI, C-arm CT with contrast agent, …)
- Collecting all data appears to require more dose (e.g. in CT)
  - Not always true, compare many low dose projections to a few high dose projections
- Cost considerations prohibit to collect all data
  - E.g. in luggage screening with static CT systems
- Hardware limitations in case of limited angle
- Defect detector pixels, over or underexposure of certain detector areas
- Missing data due to opaque objects, e.g. metal implants
- …

How to Handle Data Sparsity?

- Fill missing data with zeroes
  - Bad idea
  - Typically implicitly done by analytical algorithms
- Interpolate or extrapolate to fill unknown data points and proceed as if data were not sparse
  - No good idea
  - Useful as a first test
- Use additional knowledge about the signal (here: patient) and incorporate that a priori knowledge into signal processing
  - Optimal way to proceed
  - Typically no closed analytical solution
  - Often requires iterative data processing (data domain and a priori knowledge domain are often different)

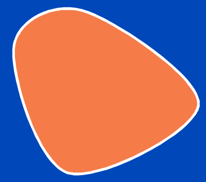
Types of a Priori Knowledge

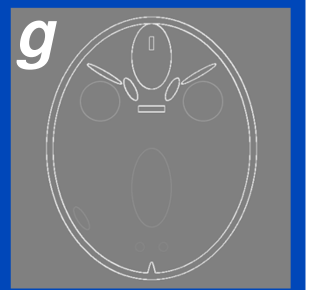


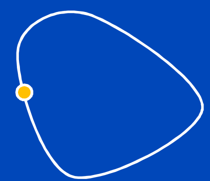
- Image gray values are restricted (e.g. to be positive)
- Patient support is finite (e.g. < 50 cm diameter)
- Image mainly consists of homogeneous areas with edges in between: gradient or edge image is sparse (i.e. consists of many pixels with value zero)
- Image is known to consist of only a few non-zero Fourier or wavelet coefficients
- Object moves from frame to frame according to a smooth motion vector field
- Object motion is quasi periodic
- Patient anatomy is known (e.g. prior to intervention) and only a few regions are expected to change (those that receive contrast agent or those that contain interventional material)
- Patient anatomy is approximately known (e.g. from an atlas)
- Artifacts are approximately known and can be predicted
- …

Sparseness Transform

- Reconstruction means solving $p = A \cdot f$.
- If measured data are sparse the linear system is underdetermined.
- To find a proper solution with less artifacts include a priori knowledge.
- Include a priori knowledge by transforming the image f such that the result g of the transformation is known to be sparse (i.e. known to have many zero entries).
- Search for the solution f to $p = A \cdot f$ that results in the most sparsest g.

$f \in$




$f =$


How to Measure Sparsity?

- By counting nonzero entries via L_0-norm:

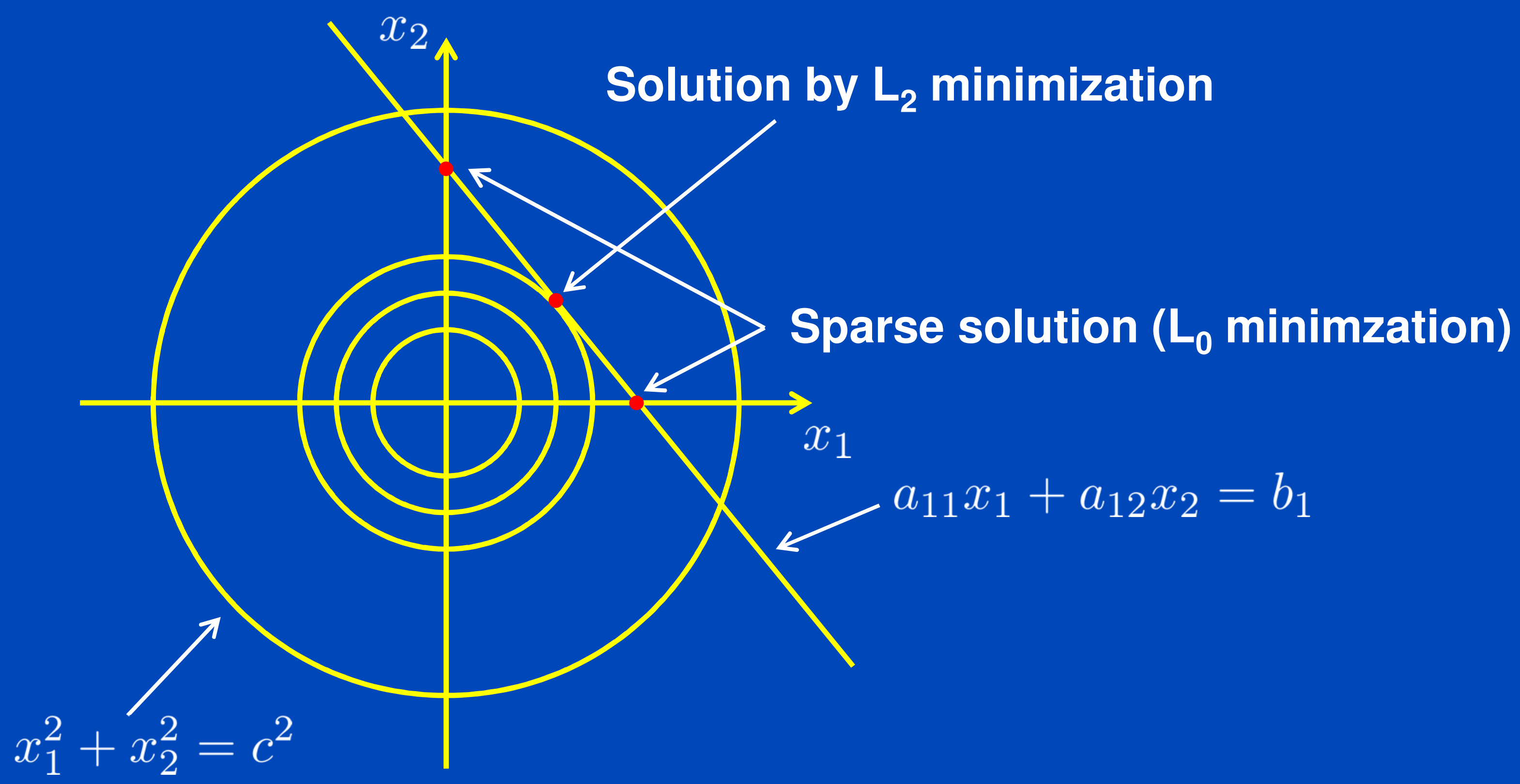
$$\|g\|_0 = \sum_i x_i^0 \text{ with } 0^0 = 0$$

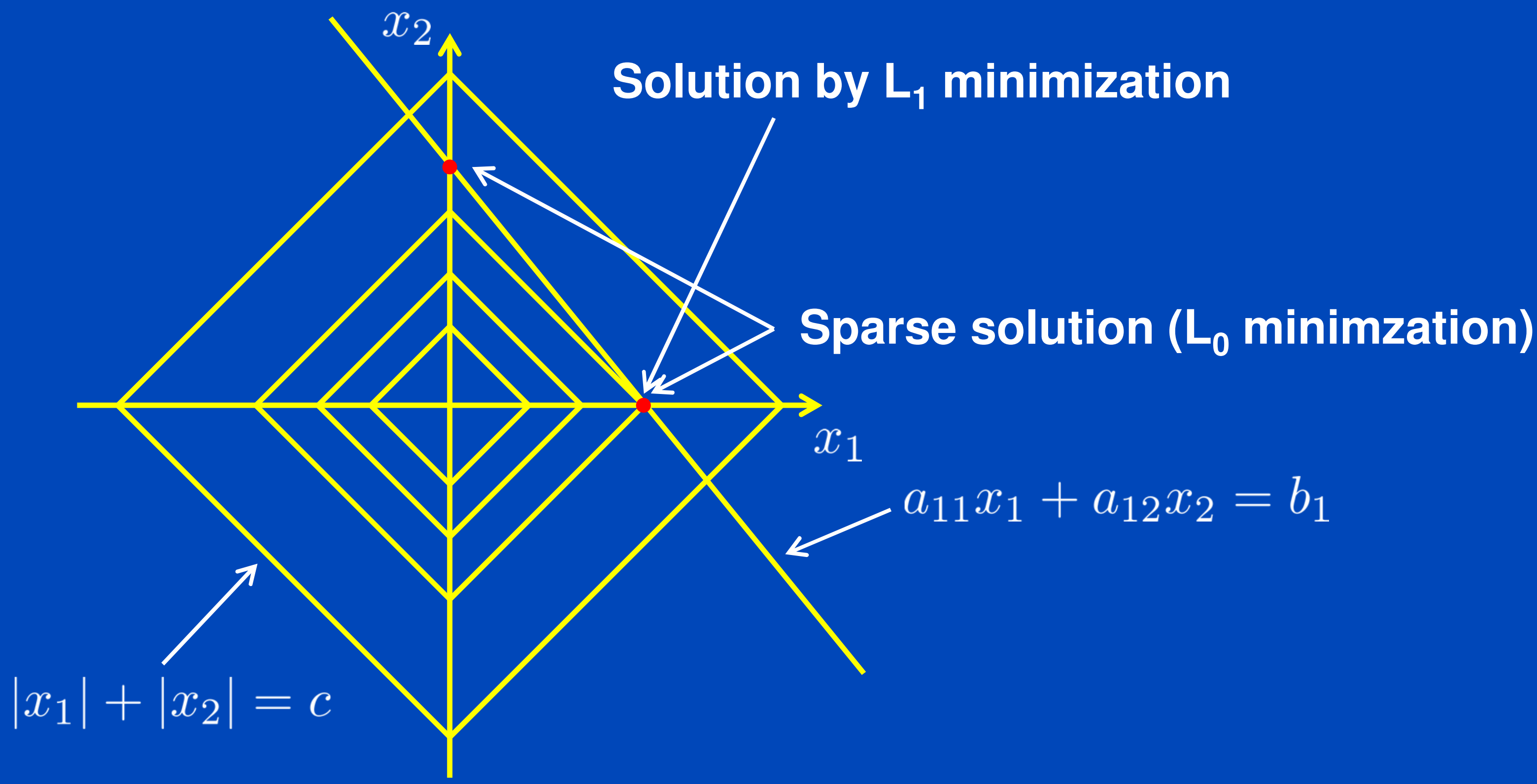
- L_0-norm is not convex. Minimization is not possible unless global search methods are used.
- Alternatives:
  - L_1-norm: not strictly convex and non-differentiable, but better approximation to L_0-norm

$$\|g\|_1 = \sum_i |x_i|$$

  - L_2-norm: strictly convex and differentiable => possible to use standard convex optimization methods:

$$\|g\|_2 = \sqrt{\sum_i x_i^2}$$


x_2
Solution by L_2 minimization
Sparse solution (L_0 minimzation)
x_1
$a_{11}x_1 + a_{12}x_2 = b_1$
$x_1^2 + x_2^2 = c^2$



x_2
Solution by L_1 minimization
Sparse solution (L_0 minimzation)
x_1
$a_{11}x_1 + a_{12}x_2 = b_1$
$|x_1| + |x_2| = c$

Simple Demonstration

True image *f* and projection data *p*

0	0	0	0
0	6	0	6
0	0	0	0

0	6	0

$L_0 = 1$, $L_1 = 6$, $L_2 = \sqrt{36}$

Three of the other solutions

1	-2	1
-2	10	-2
1	-2	1

$L_0 = 9$, $L_1 = 22$, $L_2 = \sqrt{120}$

-1	2	-1
2	2	2
-1	2	-1

$L_0 = 9$, $L_1 = 14$, $L_2 = \sqrt{24}$

0	0	0
0	5	1
0	1	-1

$L_0 = 4$, $L_1 = 8$, $L_2 = \sqrt{28}$

Total Variation (TV) Minimization

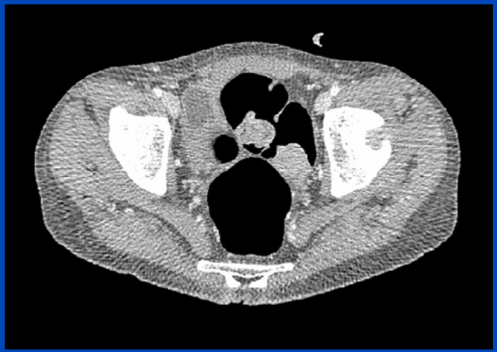
- Image gradient magnitude:

$$|\nabla f(x,y)| = \sqrt{(f(x,y)-f(x-1,y))^2+(f(x,y)-f(x,y-1))^2}$$

- Total variation (=L_1-norm of the gradient image):

$$\begin{aligned}\|f\|_{\mathrm{TV}} &= \sum_{x,y} |\nabla f(x,y)| \\ &= \sum_{x,y} \sqrt{(f(x,y)-f(x-1,y))^2+(f(x,y)-f(x,y-1))^2}\end{aligned}$$

- If applied as sparsifying transformation: search for an image with only few edges or smooth edges
- Suppresses noise in the image



Higher TV

Lower TV

$$\|f\|_{\mathrm{TV}} = \|\nabla f(\boldsymbol{r})\|_1$$

Example 1: iTV

- **Improved total variation minimization (iTV)**

IOP PUBLISHING PHYSICS IN MEDICINE AND BIOLOGY

Phys. Med. Biol. **56** (2011) 1545–1561 doi:10.1088/0031-9155/56/6/003

Improved total variation-based CT image reconstruction applied to clinical data

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Online at stacks.iop.org/PMB/56/1545

Abstract

In computed tomography there are different situations where reconstruction has to be performed with limited raw data. In the past few years it has been shown that algorithms which are based on compressed sensing theory are able to handle incomplete datasets quite well. As a cost function these algorithms use the ℓ_1-norm of the image after it has been transformed by a sparsifying transformation. This yields to an inequality-constrained convex optimization problem. Due to the large size of the optimization problem some heuristic optimization algorithms have been proposed in the past few years. The most

Two Formulations of the Same Problem

Constrained formulation

$$\min ||\nabla f(\mathbf{r})||_1 \text{ subject to } ||\mathbf{Rf}(\mathbf{r}) - \mathbf{p}||_2^2 < \epsilon$$

unknown

Unconstrained formulation

$$\min ||\nabla f(\mathbf{r})||_1 + \mu ||\mathbf{Rf}(\mathbf{r}) - \mathbf{p}||_2^2$$

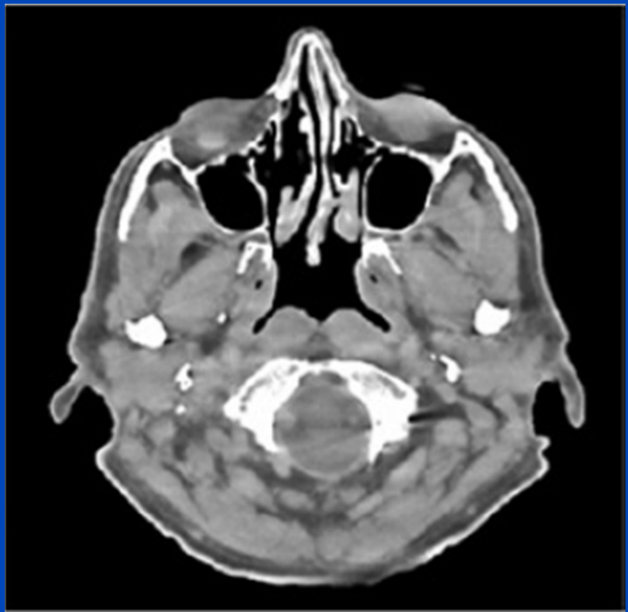
unknown

How to choose ε or μ?

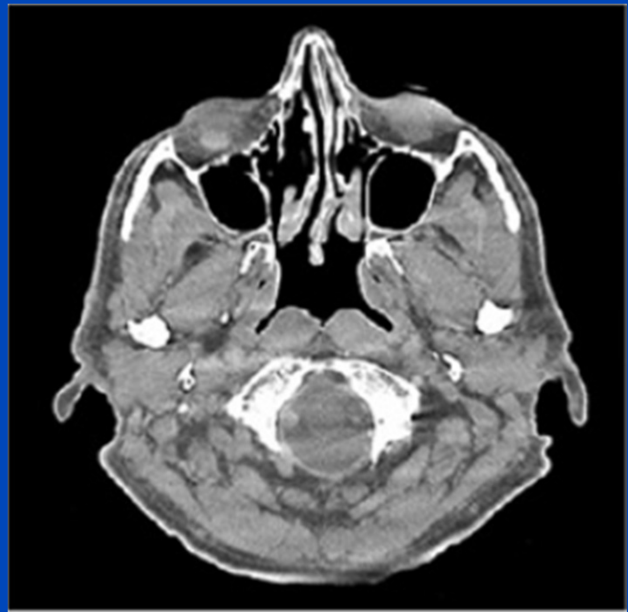
- For every value ε there exists a value of μ which yields the same solution of the problem.
- There exists a value $\epsilon_{\mathrm{opt}} > 0$, which represents the best possible rawdata fidelity of the reconstructed image.

Important

- Only solutions close to $\epsilon_{\mathrm{opt}} > 0$ really represent the physically measured image content.
- Otherwise the influence of the regularization function will become too strong.



$\epsilon > \epsilon_{\mathrm{opt}}$



$\epsilon \approx \epsilon_{\mathrm{opt}}$

iTV Algorithm

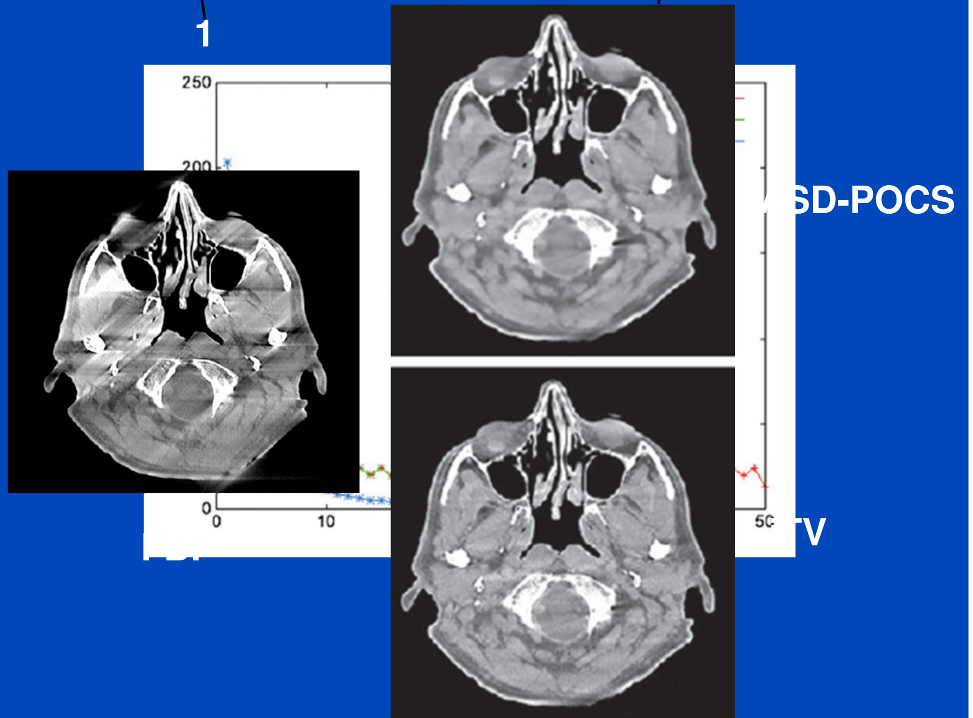
- Solves the constrained problem
- Reaches a solution close to $\epsilon_{\mathrm{opt}} > 0$ automatically.
- Leads to an approximative solution of the constrained optimization problem

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

iTV Algorithm

- **Constrained optimization problem**
- **Solve 1 with gradient descent and 2 with SART separately.**
- **iTV adapts both steps automatically to ensure result at lowest value of ε.**
- **Reconstructed image is consistent with projections, no oversmoothing. No a priori knowledge of ε.**

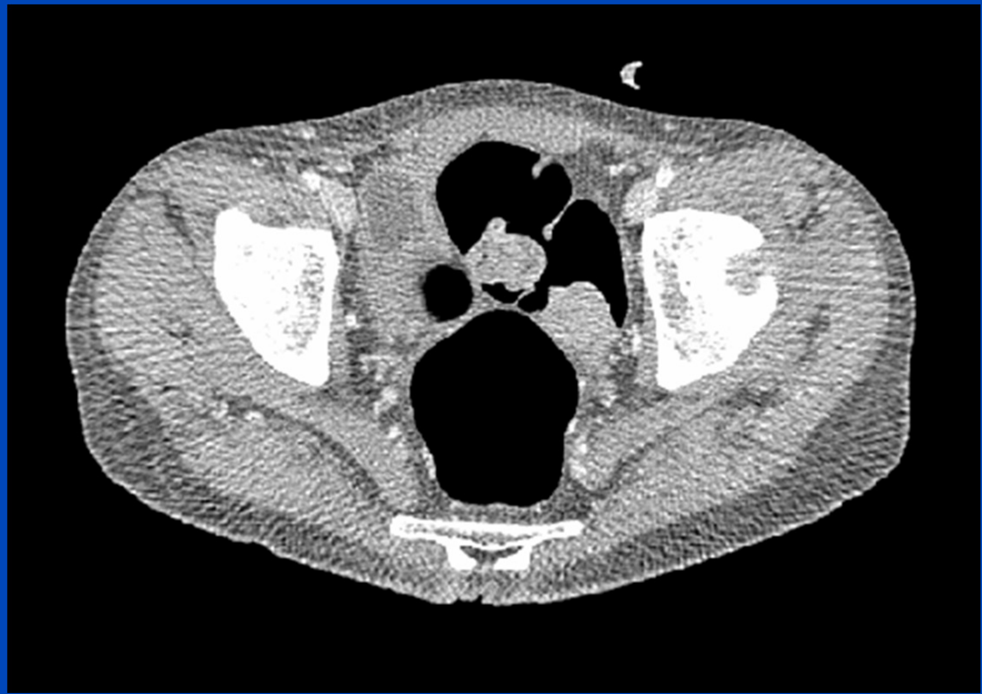
$$\min \|\nabla f(r)\|_1 \quad \text{subject to} \quad \|\mathrm{R} f(r) - p\|_2^2 < \epsilon$$



Sidky E Y, Pan X et al, 2009. ***Enhanced imaging of microcalcifications in digital breast tomosynthesis through improved image reconstruction algorithms*** **Med.Phys. 36 4920–32**

Sparse Projections

FBP 128 Projections



C = 0 HU   W = 400 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Sparse Projections

iTV 128 Projections

C = 0 HU   W = 400 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Sparse Projections

Reference

C = 0 HU   W = 400 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Metal Implants

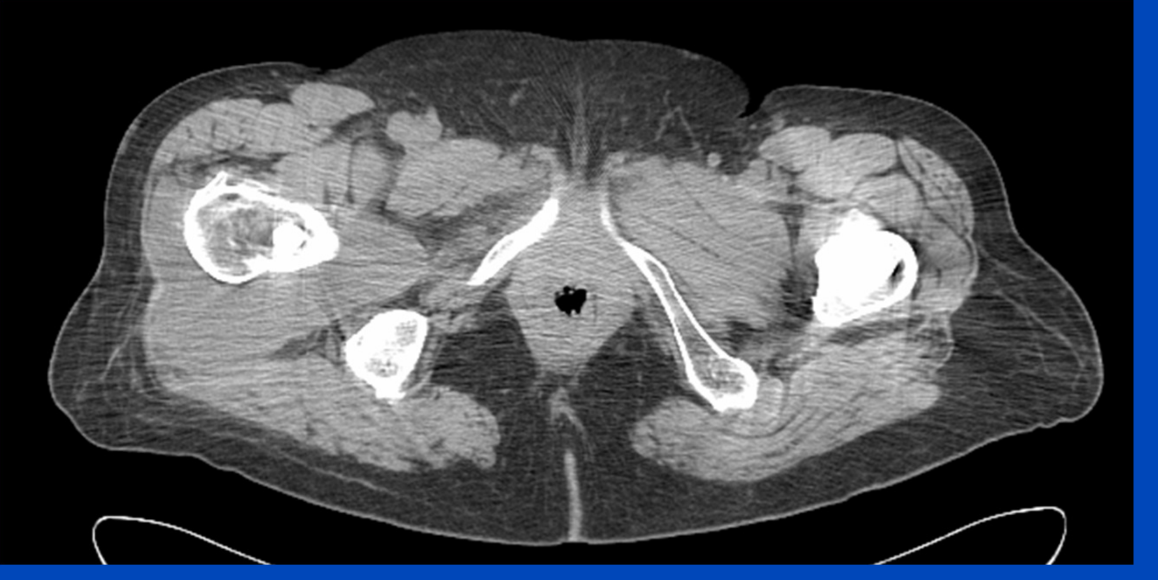
FBP

C = 0 HU   W = 400 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Metal Implants

iTV



C = 0 HU   W = 400 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Limited Angle Tomography

FBP 135°

C = 0 HU   W = 500 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Limited Angle Tomography

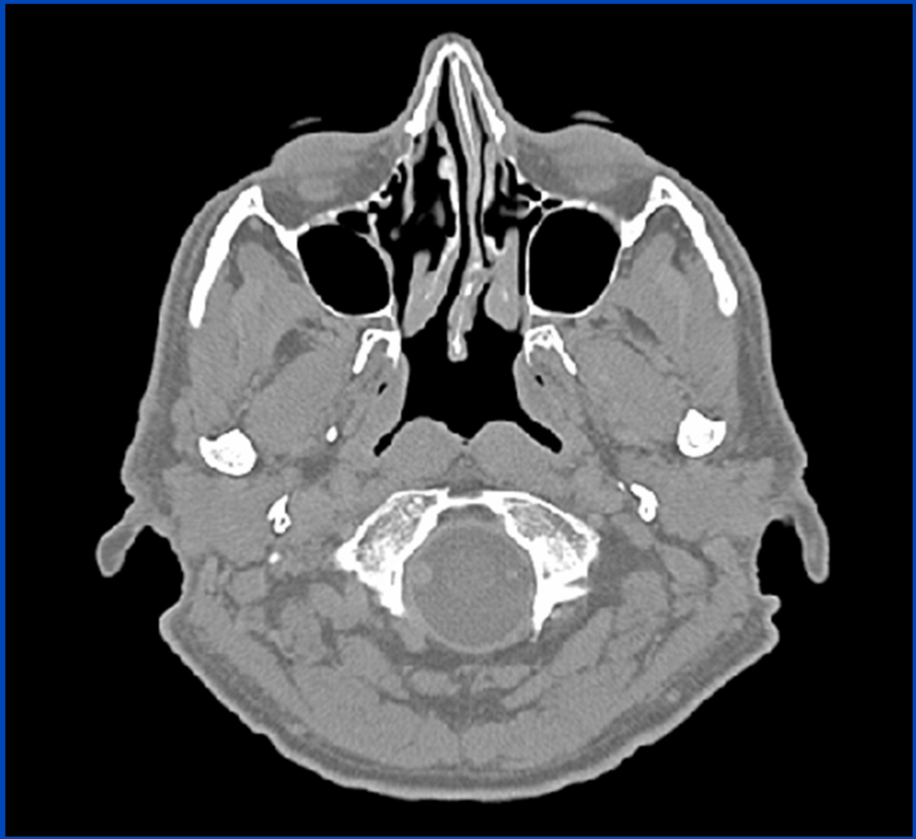
iTV 135°

C = 0 HU   W = 500 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

Limited Angle Tomography

Ground Truth

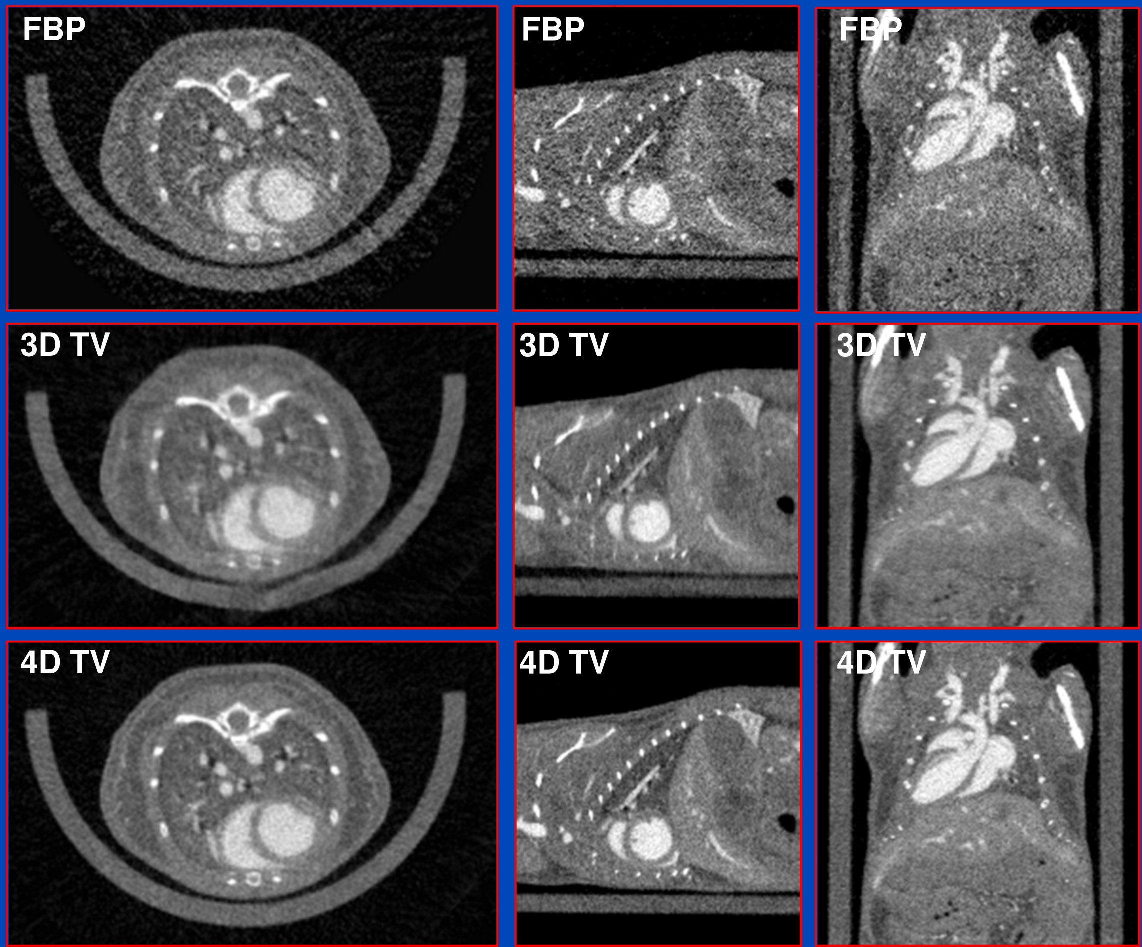


C = 0 HU   W = 500 HU

Ritschl, L. ; Bergner, F. ; Fleischmann, C. ; Kachelrieß, M. : Improved total variation based CT image reconstruction applied to clinical data. PMB 56, 2011.

The iTV Approach is Extendable to More Complex Problems

Axial | Sagittal | Coronal



$$\nabla_{\boldsymbol{r},t} f(\boldsymbol{r},t) = \begin{pmatrix} \alpha \cdot \frac{1}{\Delta x}\left(f_{x,y,z,t} - f_{x-1,y,z,t}\right) \\ \alpha \cdot \frac{1}{\Delta y}\left(f_{x,y,z,t} - f_{x,y-1,z,t}\right) \\ \alpha \cdot \frac{1}{\Delta z}\left(f_{x,y,z,t} - f_{x,y,z-1,t}\right) \\ (1-\alpha) \cdot \frac{1}{\Delta t}\left(f_{x,y,z,t} - f_{x,y,z,t-1}\right) \end{pmatrix}$$

Cardiac Gating : ΔC=10% Image window: C=0 HU / W=1200 HU

Ritschl, L. ; Sawall, S. ; Knaup, M. ; Hess, A. ; Kachelrieß, M. : *Iterative 4D cardiac micro-CT image reconstruction using an adaptive spatio-temporal sparsity prior.* PMB 57, 2012.

Example 2: acMoCo

- **Motion-compensated (MoCo) image reconstruction for image-guided radiation therapy (IGRT)**

Self-adapting cyclic registration for motion-compensated cone-beam CT in image-guided radiation therapy

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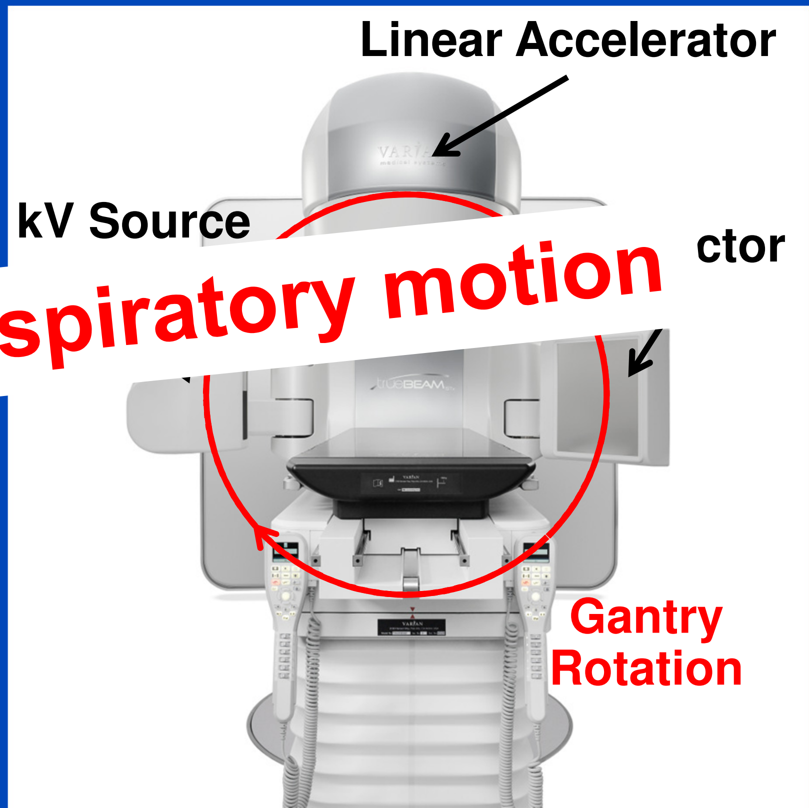
Purpose: In image-guided radiation therapy an additional kV imaging system next to the linear particle accelerator provides information for an accurate patient positioning. However, the acquisition time of the system is much longer than the patient's breathing cycle due to the low gantry rotation speed. Our purpose is a cyclic registration in the context of motion-compensated image reconstruction that provides high quality respiratory-correlated 4D volumes for on-board flat panel detector cone-beam CT scans.
Methods: Based on the small motion assumption, widely used within registration algorithms, a strategy is developed for motion estimation. In this strategy temporal restrictions are incorporated, for example, the cyclic motion patterns of respiration. The resultant cyclic registration method is to show less sensitivity on image artifacts, in particular on artifacts due to projection data sparsification. Using a new cyclic registration method a motion estimation is performed on respiratory-correlated reconstructions, and the obtained motion vector fields are used for motion compensation.

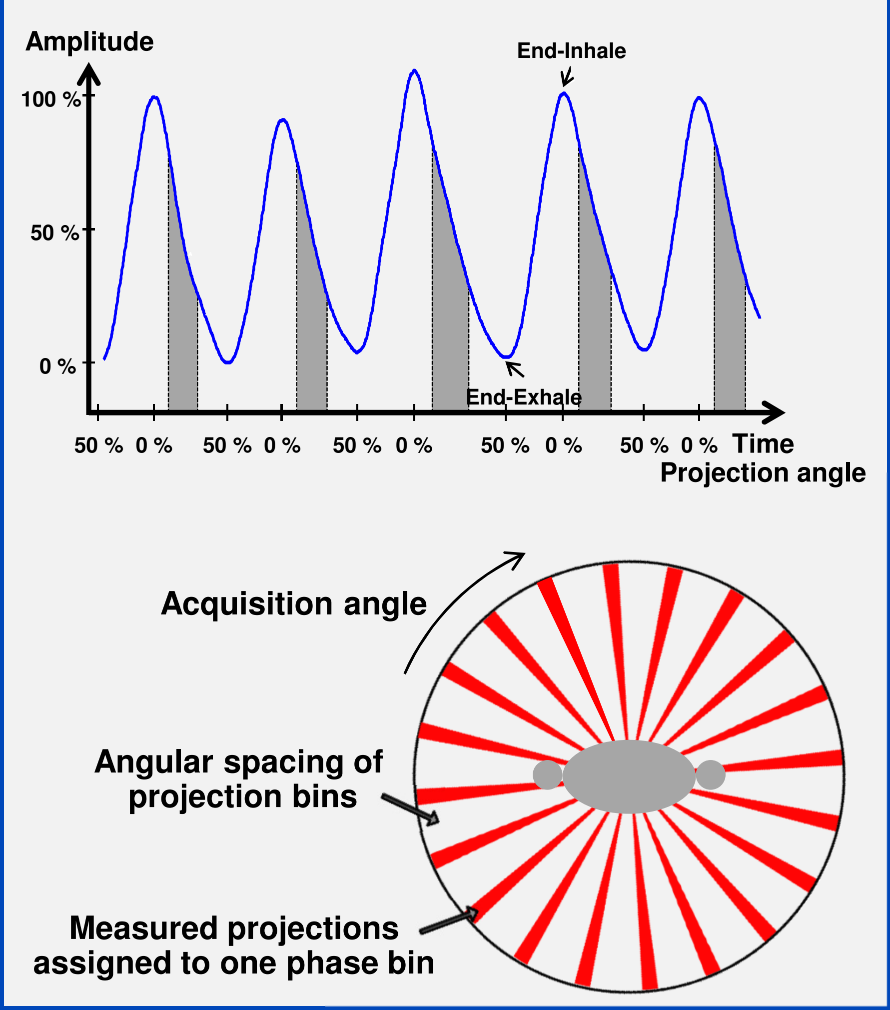
Slowly Rotating CBCT Devices

- Image-guided radiation therapy (IGRT)
  - CBCT imaging unit mounted on gantry of a LINAC treatment system
  - E.g. used for patient positioni
- M
  s per second
  - Much slower than clinical CT devices (60 s and more vs. about 0.28 s per rotation)
- Breathing cycle about 2 to 5 seconds
  - i.e. 12 to 30 respirations per minute (rpm) and thus per scan



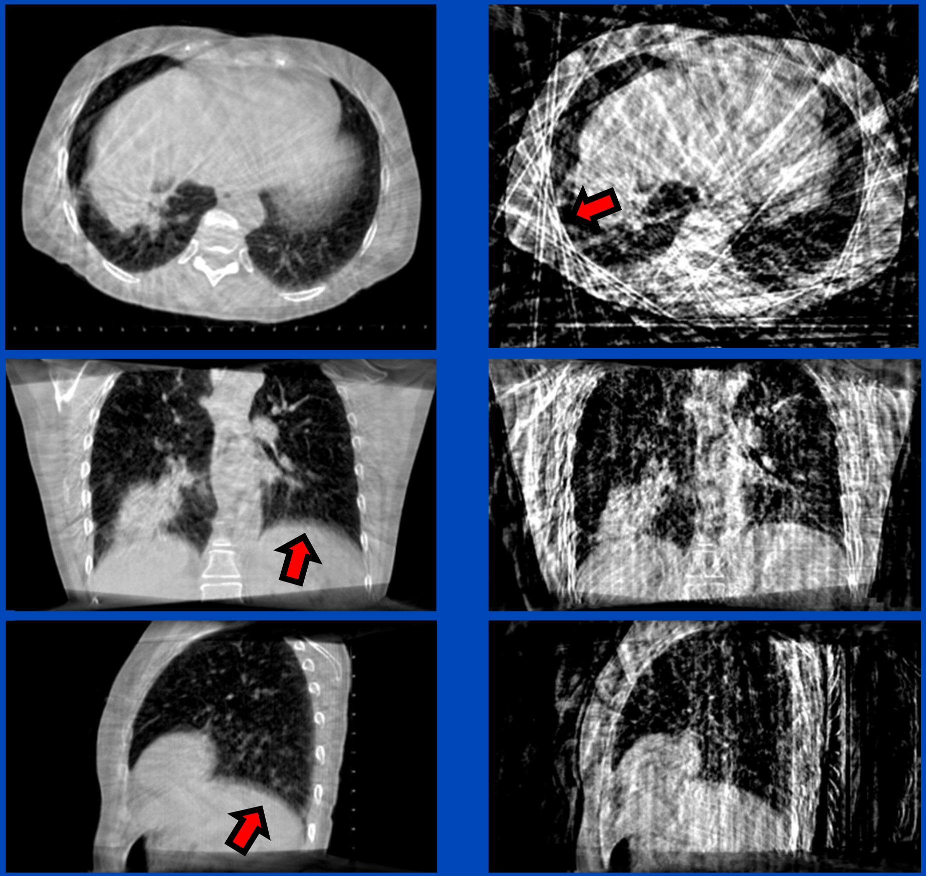
Task: Account for respiratory motion

Retrospective Gating



Without gating (3D): Motion artifacts

With gating (4D): Sparse-view artifacts

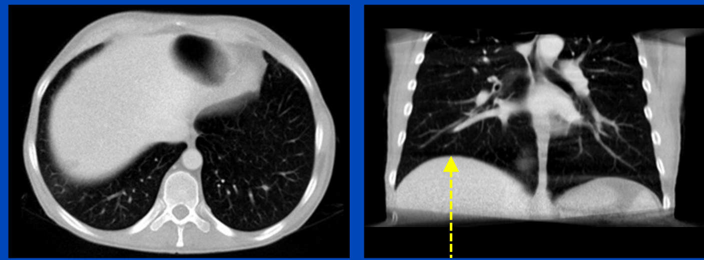


Motion Compensation (MoCo)

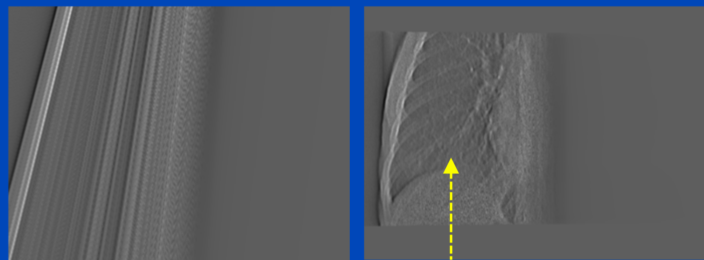
- **Use all projection data for each phase to be reconstructed**
  - Even those of other phase bins
  - Compensate for motion using motion vector fields (MVFs)
  - In our case MVFs are estimated from phase-correlated (gated) reconstructions
- **Backproject-then-warp**
  - Backproject sparse data along straight lines, warp with respect to the MVFs, and superimpose warped backprojections of all sparse data
  - Projection data p, phase-correlated reconstruction operator $\mathrm{X}_{\mathrm{PCF}}^{-1}$, MVF T_j^i from phase bin j to phase bin i

$$f_{\mathrm{MoCo}(i)} := \sum_j \left(\mathrm{X}_{\mathrm{PCF}(j)}^{-1}\, p\right) \circ \mathrm{T}_j^i$$

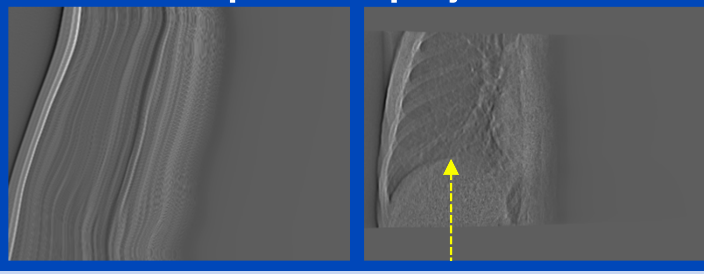
Ground truth <u>in end-exhale</u>



Backprojection on (straight) acquisition lines of a projection acquired <u>in end-inhale</u>

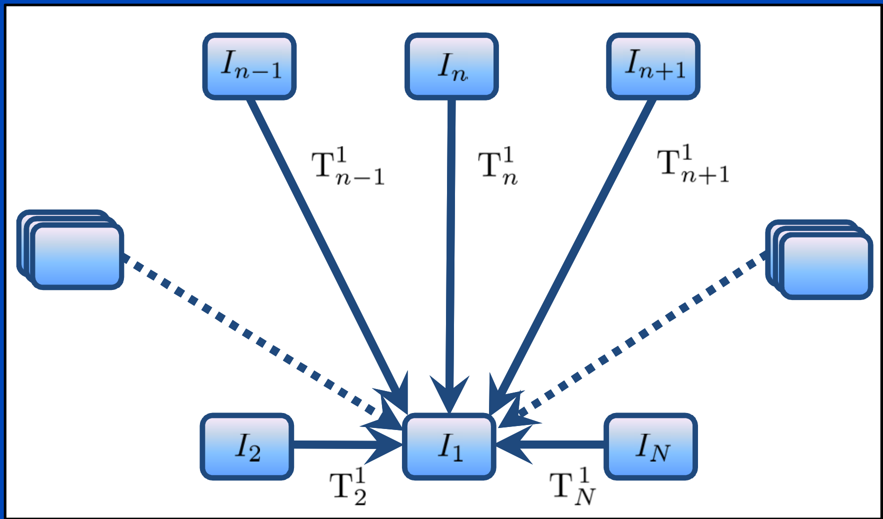


Warped backprojection



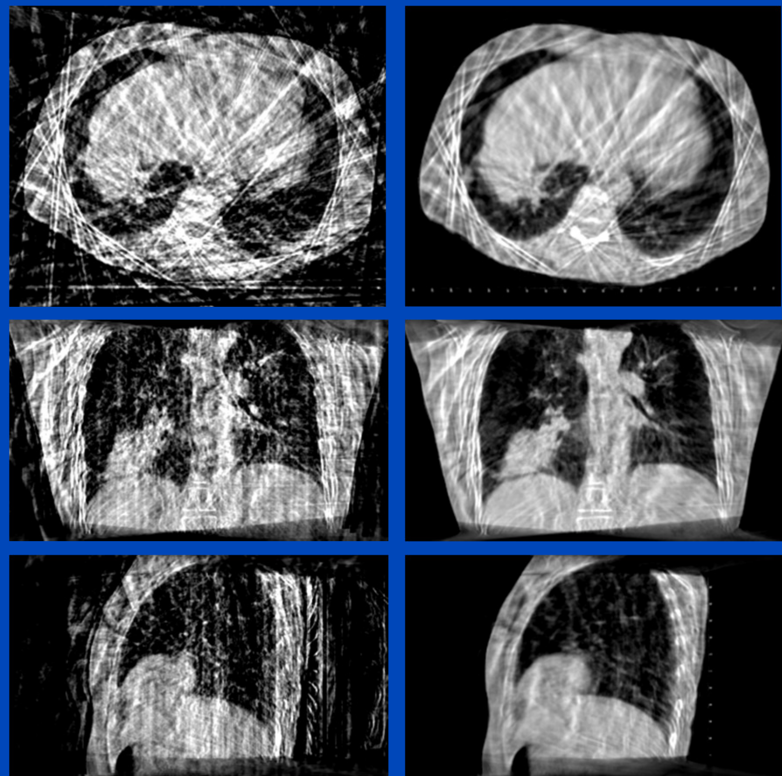
A Standard Motion Estimation and Compensation Approach (sMoCo)

- Motion estimation via standard 3D-3D registration



- Has to be repeated for each reconstructed phase

Gated 4D CBCT | sMoCo

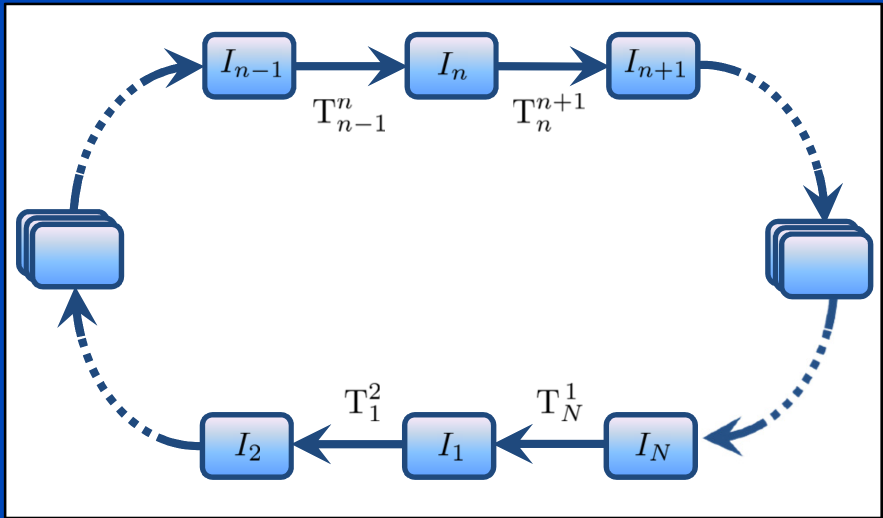


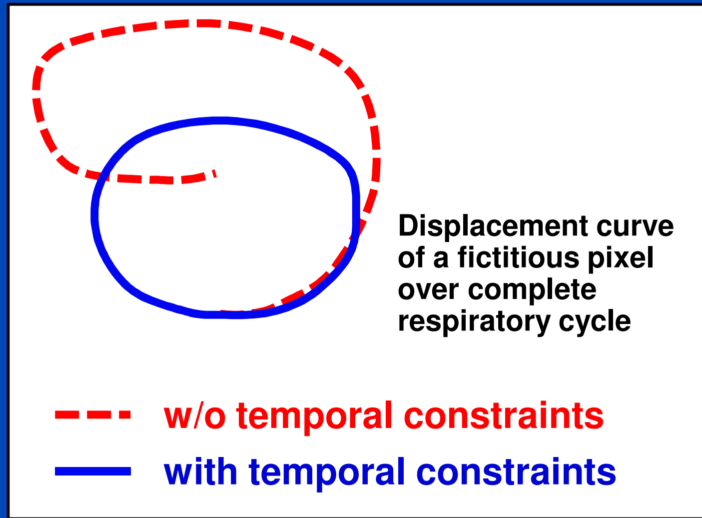
- Streak artifacts from gated reconstructions propagate into sMoCo results

Li et al., "*Enhanced 4D cone–beam CT with inter–phase motion model*," Med. Phys. 51(9), 3688–3695 (2007).

A Cyclic Motion Estimation and Compensation Approach

- Motion estimation only between adjacent phases
  - All other MVFs given by concatenation



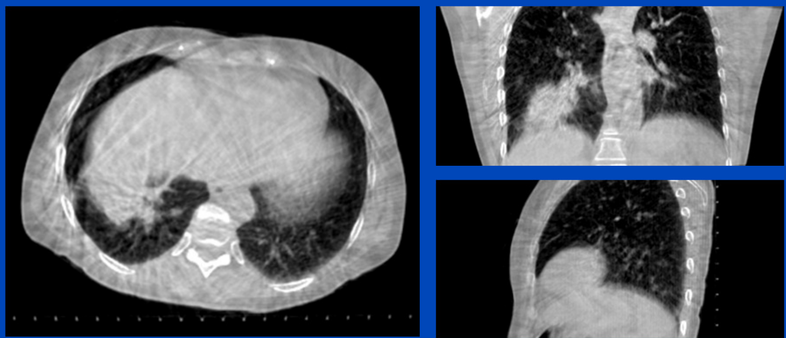


- Incorporate additional knowledge
  - A priori knowledge of quasi periodic breathing pattern
  - Non-cyclic motion is penalized
  - Error propagation due to concatenation is reduced

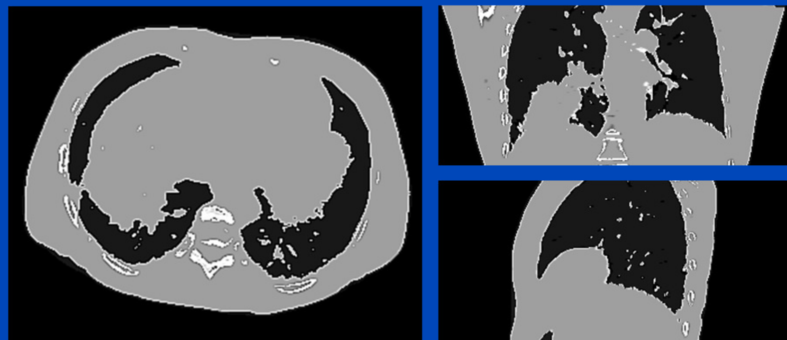
Brehm, Paysan, Oelhafen, Kunz, and Kachelrieß, "Self-adapting cyclic registration for motion-compensated cone-beam CT in image-guided radiation therapy," Med. Phys. 39(12), 7603-7618 (2012).
Reported at CT-Meeting 2012, MIC 2012, RSNA 2012.

Angular Sampling Artifact Model

- Create second series of images with sparse-view artifacts but without breathing motion
- Eliminate breathing motion information
  - Threshold-based segmentation of 3D CBCT
- Simulate measurement and reconstruction process
  - Forward projection of segmented image
  - Backprojection at same angles as for gated 4D CBCT



3D CBCT

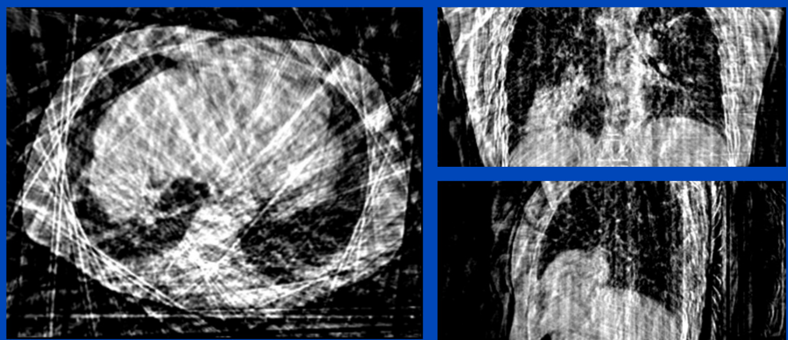


Segmented Image

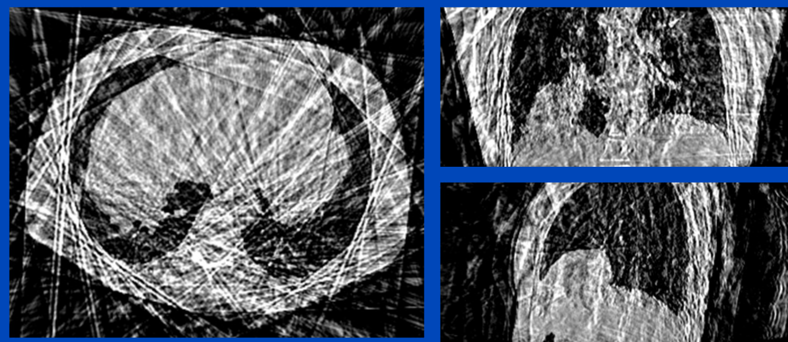
C = -200 HU, *W* = 1400 HU

Angular Sampling Artifact Model

- Create second series of images with sparse-view artifacts but without breathing motion
- Eliminate breathing motion information
  - Threshold-based segmentation of 3D CBCT
- Simulate measurement and reconstruction process
  - Forward projection of segmented image
  - Backprojection at same angles as for gated 4D CBCT



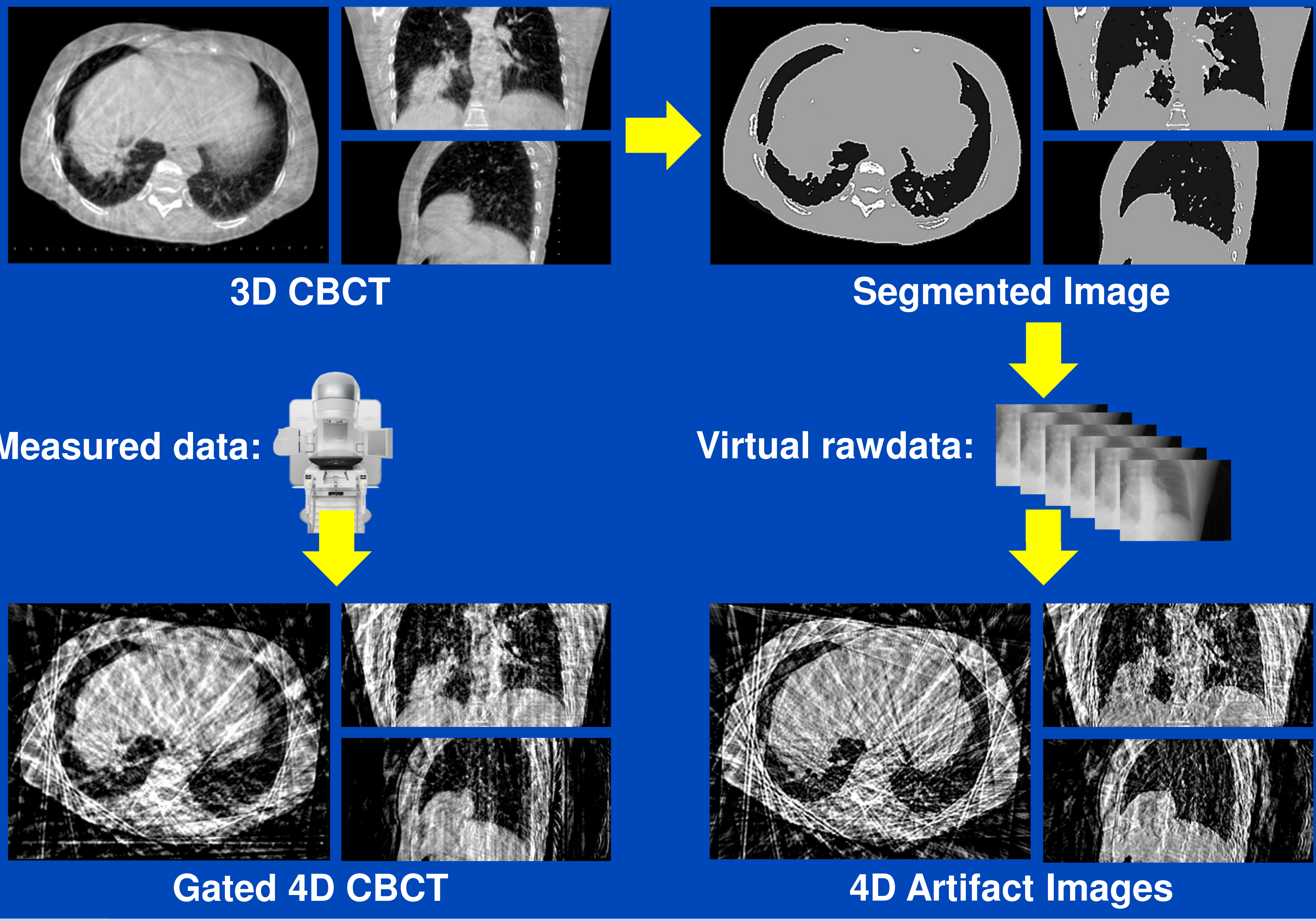
Gated 4D CBCT



4D Artifact Images

C = -200 HU, *W* = 1400 HU

Angular Sampling Artifact Model

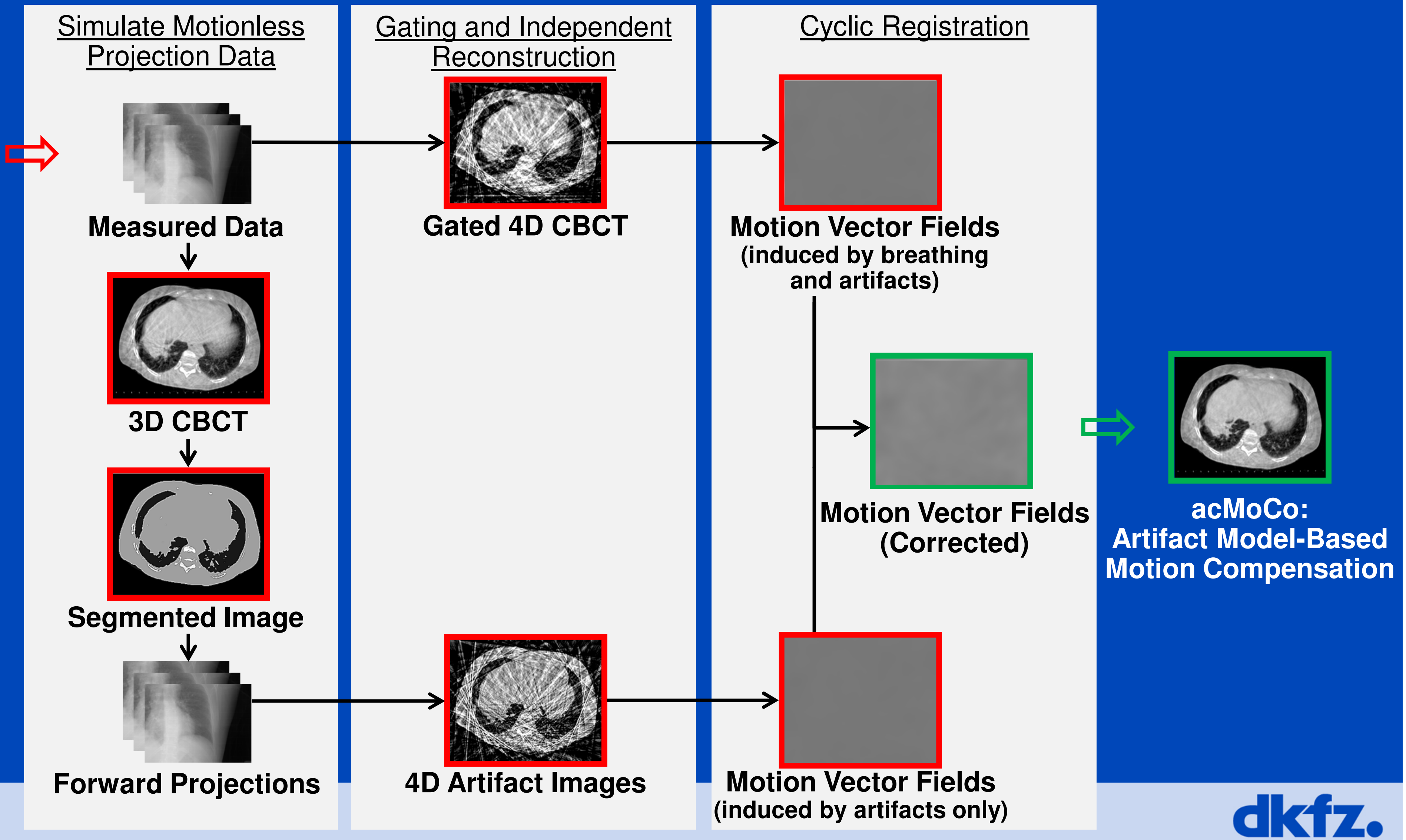


C = -200 HU, W = 1400 HU

VARIAN medical systems

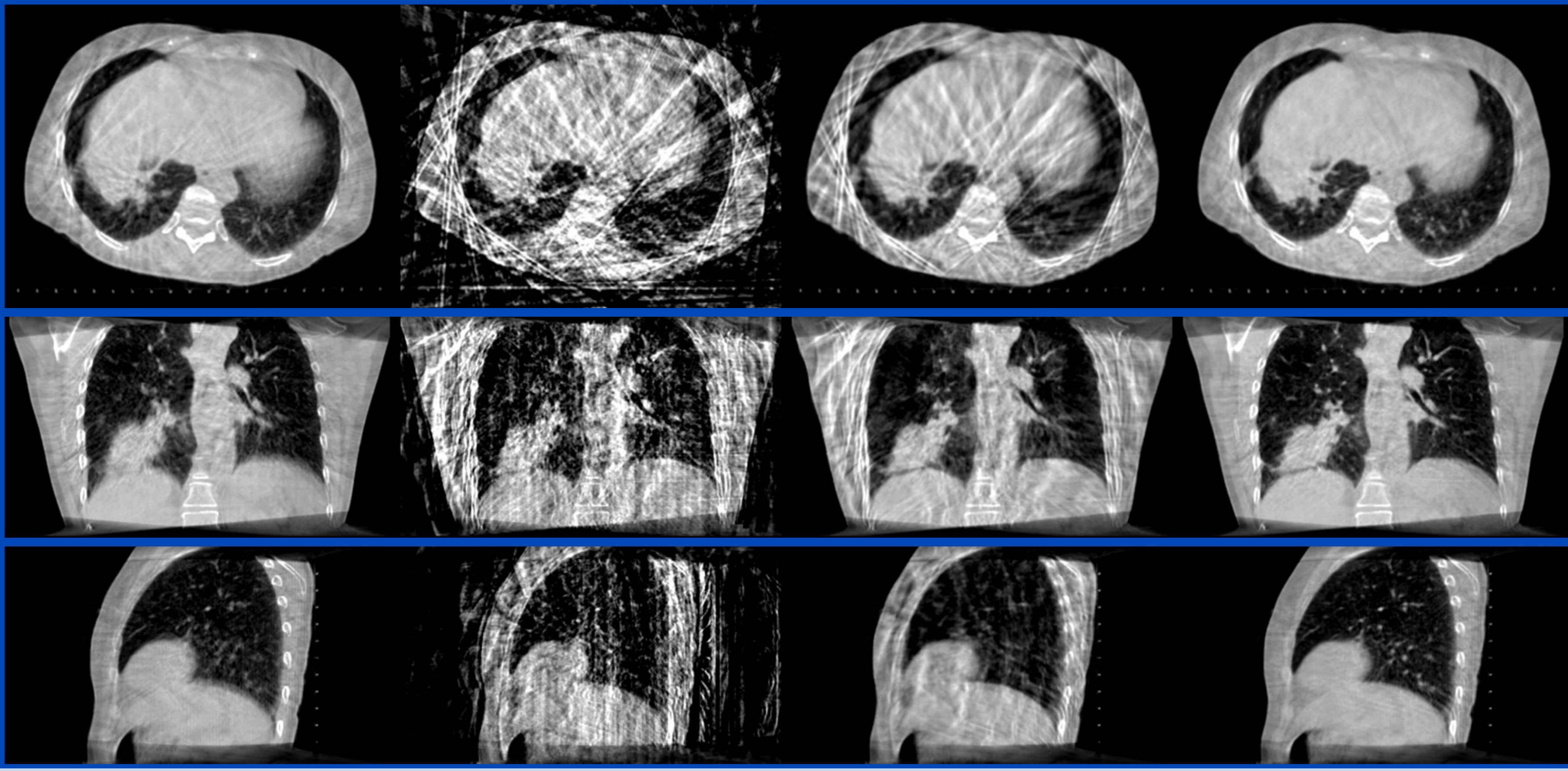
dkfz.

Motion Estimation using an Patient-Specific Artifact Model



Patient Data – Results

3D CBCT	Gated 4D CBCT	sMoCo	aMoCo
Standard	Conventional Phase-Correlated	Standard Motion Compensation	Artifact Model-Based Motion Compensation



C = -200 HU, *W* = 1400 HU

Example 3: Pridict

- **Tomographic fluoroscopy at conventional dose for interventional imaging: prior image dynamic interventional CT**

Eur Radiol (2013) 23:1669–1677
DOI 10.1007/s00330-012-2761-2

INTERVENTIONAL

Real-time X-ray-based 4D image guidance of minimally invasive interventions

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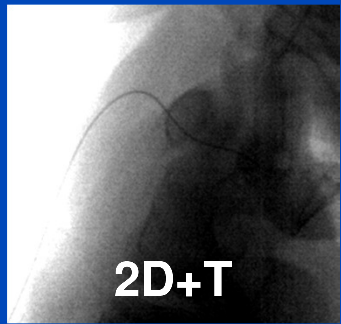
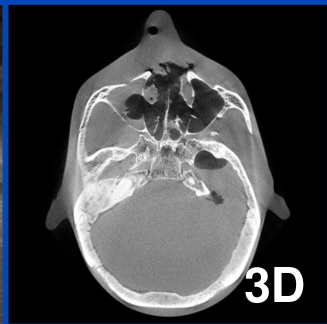
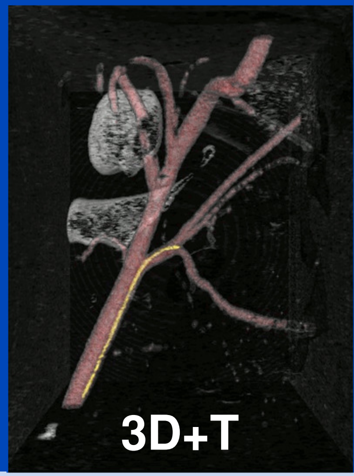
Abstract
Objective A new technology is introduced that enables real-time 4D (three spatial dimensions plus time) X-ray guidance for vascular catheter interventions with acceptable levels of ionising radiation.
Methods The enabling technology is a combination of low-dose tomographic data acquisition with novel compressed sensing reconstruction and use of prior image information. It was implemented in a prototype set-up consisting of a gantry-based flat detector system. In pigs (n=5) angiographic interventions were simulated. Radiation dosage on a per time base was compared with the "gold standard" of X-ray projection imaging.
Results Contrary to current image guidance methods that lack permanent 4D updates, the spatial position of interventional instruments could be resolved in continuous, spatial 4D guidance; the movement of the guide wire as well as the expansion of stents could be precisely tracked in 3D angiographic road maps. Dose rate was 23.8 µGy/s, similar to biplane standard angiographic fluoroscopy, which has a dose rate of 20.6 µGy/s.
Conclusion Real-time 4D X-ray image-guidance with ac-

Interventional Radiology

- **Interventional radiology:**
  - Minimally invasive interventions guided by x-ray imaging techniques
  - C-arm systems
- **Projective fluoroscopy:**
  - 2D projections
  - Position of interventional material is often ambiguous.
  - To resolve ambiguities trial-and-error approaches are applied or a 3D volume has to be acquired.
- **Low dose tomographic fluoroscopy:**
  - 3D volumes, temporally resolved
  - For clinical acceptance the dose should be limited to the same level as that of projective fluoroscopy.

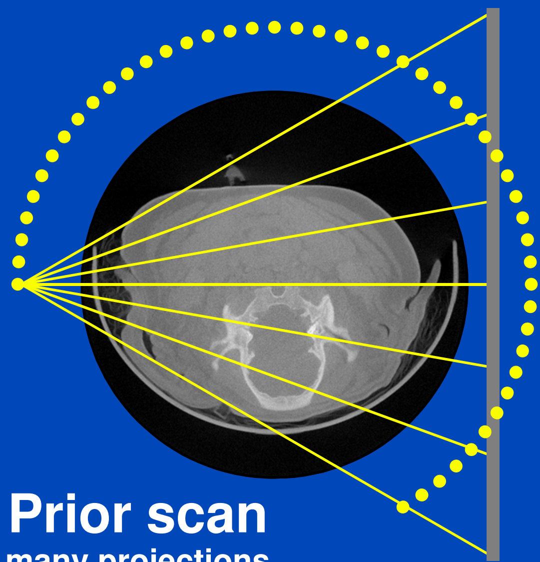




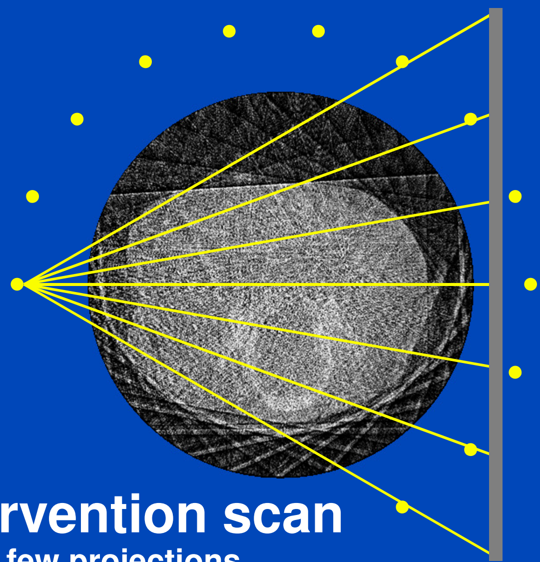


Realization of Tomographic Fluoroscopy

- Low dose by:
  - Low tube current
  - Very few projections (pulsed mode)
- Advantages of intervention guidance:
  - Repetitive scanning of the same body region.
  - Interventional materials are fine structures (few voxels) of high contrast (metal).

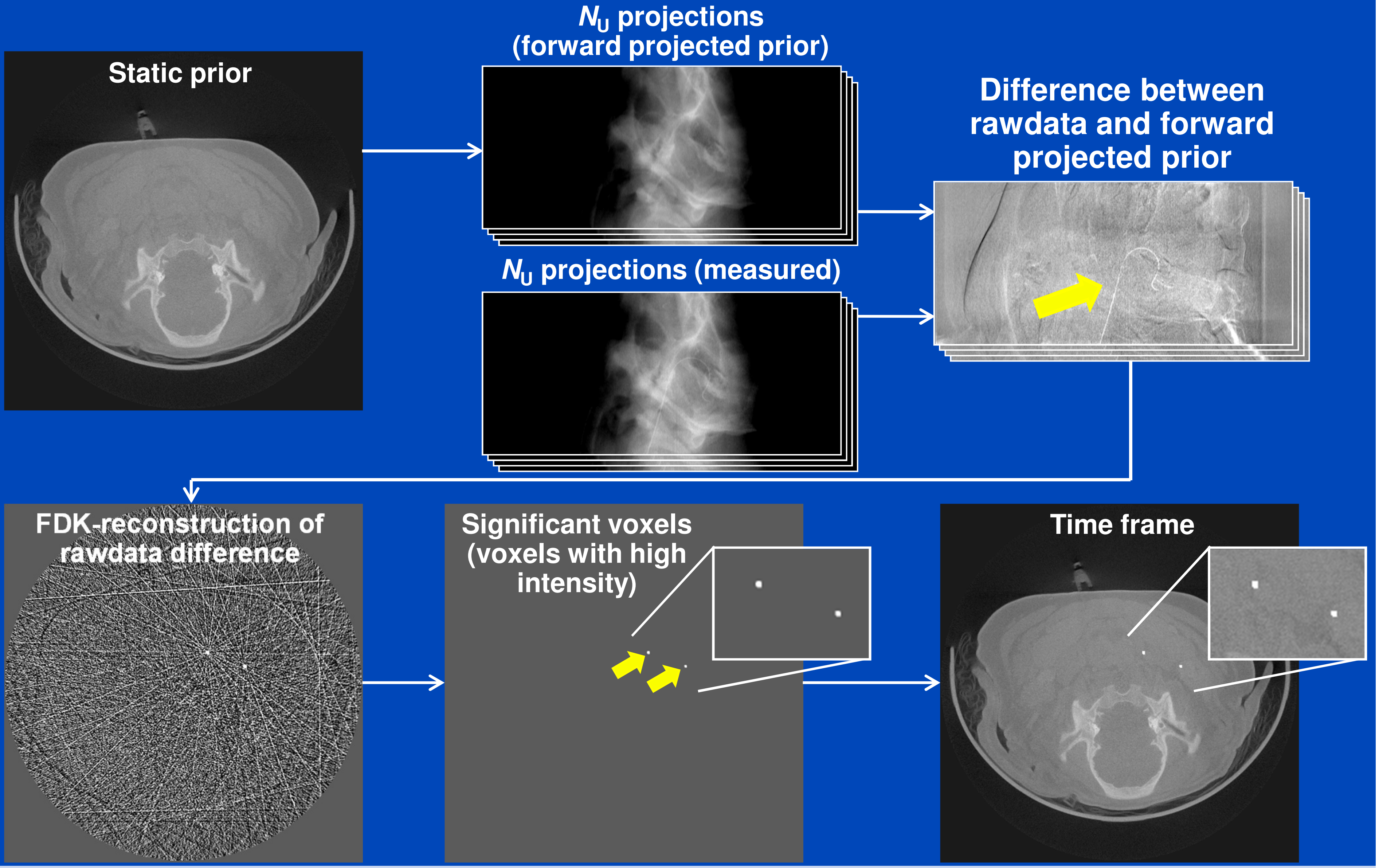


Prior scan
many projections



Intervention scan
few projections

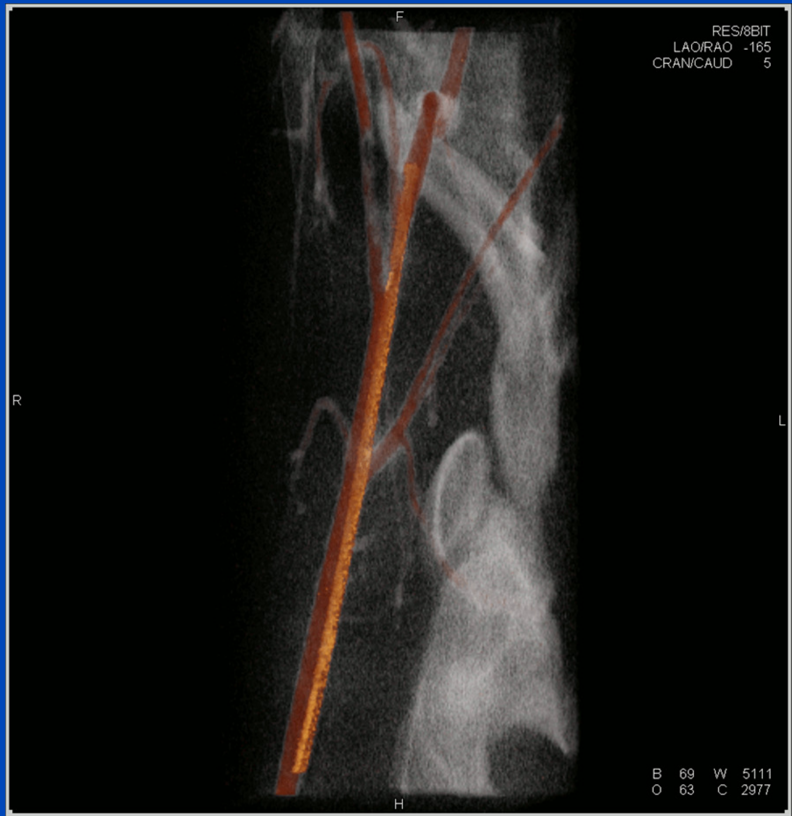
PrIDICT-Algorithm[1]



[1] J. Kuntz, R. Gupta, S.O. Schönberg, W. Semmler, M. Kachelrieß, and S. Bartling, “Real-time x-ray-based 4D image guidance of minimally invasive interventions”, Eur. Radiol., 23(6): 1669-1677, Jun. 2013.

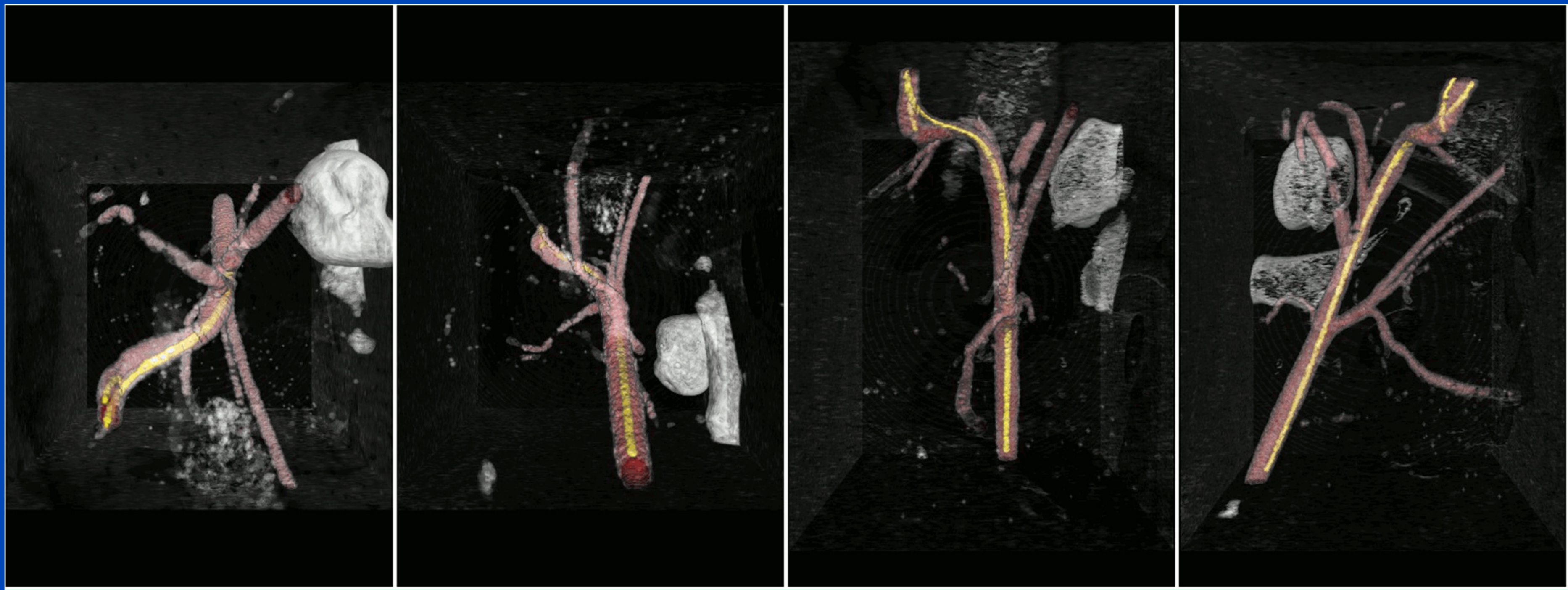
3D+T Fluoroscopy at 2D+T Dose

Stent expansion in pig carotis with angiographic roadmap overly



3D+T Fluoroscopy at 2D+T Dose

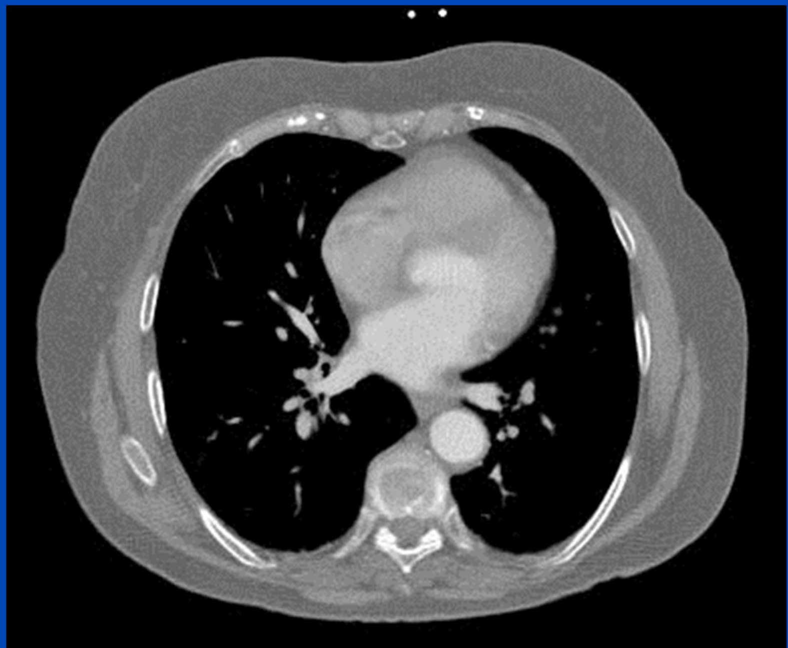
Guide wire in pig carotis with angiographic roadmap overly



Summary

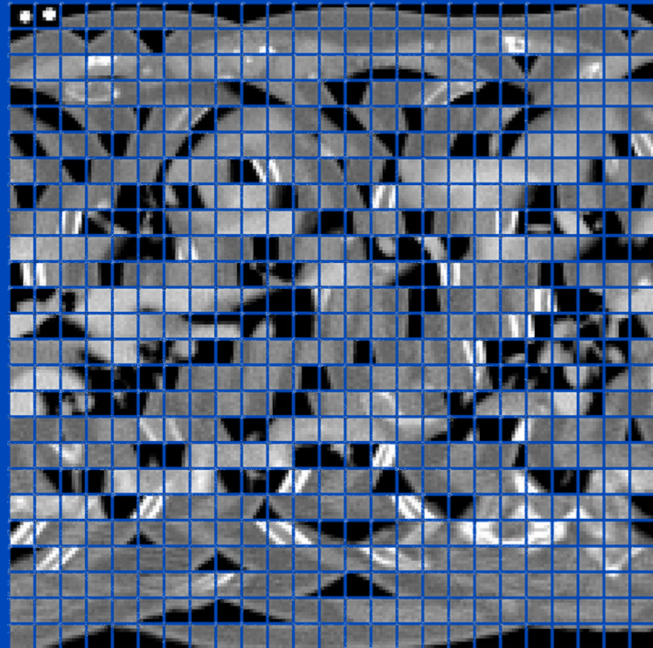
- Modern iterative reconstruction concepts (among those is compressed sensing) allow effectively to incorporate a priori knowledge.
- Data sparsity can be well handled, if priors are well chosen.
- The concepts of how to handle data sparsity are quite manifold.

FBP



C/W = 0/1000

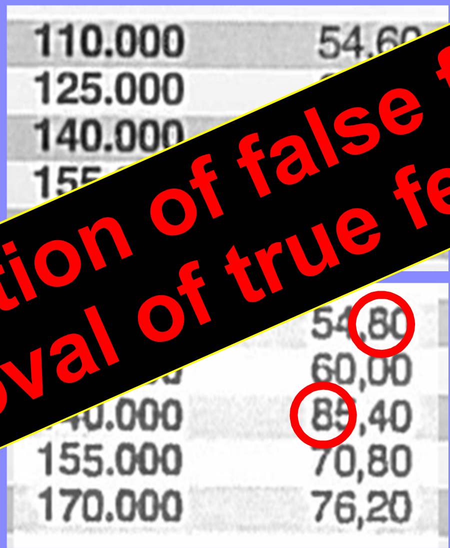
Learned dictionary



C/W = 0/1000

Xerox Compression Problem

True TIFF scan



Detail 1 | Detail 2 | Detail 3

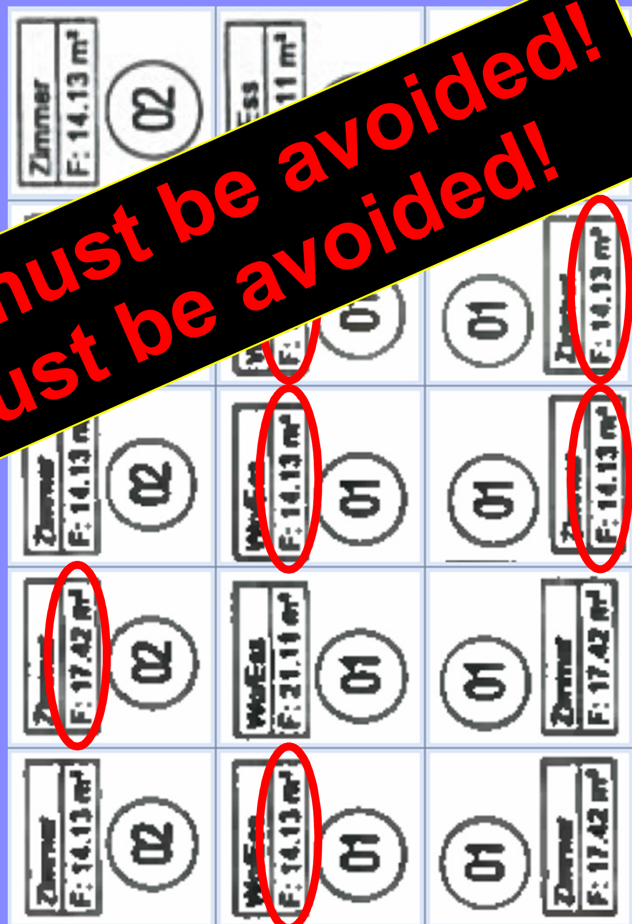
True TIFF scan

Xerox WorkC… 7535 P…

…scan 1

Xerox WorkCentre 7536 PDF scan 2

Xerox WorkCentre 7536 PDF scan 3



Introduction of false features must be avoided!
Removal of true features must be avoided!

May related problems also occur in medical image reconstruction or restoration?
E.g. when using priors based on non local means (NLM)?

Thank You!

This presentation will soon be available at www.dkfz.de/ct.

Parts of the reconstruction software were provided by RayConStruct® GmbH, Nürnberg, Germany.