Accurate Reconstruction of X-Ray Spectra in CT from Simple Transmission Measurements

Carsten Leinweber, Joscha Maier, and Marc Kachelrieß

German Cancer Research Center (DKFZ), Heidelberg, Germany
Introduction

• CT applications that require accurate knowledge of the emitted or detected spectrum:
  – Organ dose estimation
  – Beam hardening correction
  – Dual energy decomposition
  – K-edge imaging
  – Quantitative perfusion measurements
  – ...

• Existing methods:
  – Semi-analytic models
  – Monte-Carlo simulation
  – Spectroscopy
  – Compton scattering
  – Transmission measurements (direct, simple, no extra hardware)
  – ...

![Dose Estimation](image1)

![Beam Hardening Correction](image2)
Materials and Methods

Spectrum Reconstruction from Transmission Measurements

• Lambert-Beer law:

\[ \tau_m = \frac{N_m}{N_0} = \sum_{b=1}^{B} e^{-\mu_{mb} d_m w_b} \]

• Problem:

“Given \( \tau \) for different (known) combinations of \( \mu(E) \) and \( d \), reconstruct \( w(E) \).”

• Methods:
  – Few parameter modelling
  – Neural networks
  – Expectation maximization (EM)
  – Truncated singular value decomposition (TSVD)
  – New: PTSVD

**Materials and Methods**

**Truncated Singular Value Decomposition (TSVD)**

- Discretized Lambert-Beer law in matrix notation:
  \[
  \tau_m = \sum_{b=1}^{B} a_{mb} w_b \quad \rightarrow \quad \tau = A \cdot w
  \]

- Minimize the least square difference
  \[
  w = \arg \min \left\| A \cdot w - \tau \right\|_2^2 \quad \rightarrow \quad w = A^+ \cdot \tau
  \]

- Calculation of the pseudo-inverse \( A^+ \)
  - Decompose \( A \) into orthonormal basis with help of SVD:
    \[
    A = \sum_{b=1}^{B} u_b \cdot s_b v_b^T
    \]
  - Truncate \( A^+ \) to the highest \( R \) singular values:
    \[
    w = \sum_{b=1}^{R} \left( v_b \cdot \frac{u_b^T}{s_b} \right) \cdot \tau \quad \text{with} \quad R \leq B
    \]
Materials and Methods

Prior Truncated Singular Value Decomposition (PTSVD)

- Minimize the weighted least square difference with help of TSVD to obtain the low frequent solution from range:

\[ w_R = \arg\min_w \| A \cdot w - \tau \|_W^2 \quad \text{with} \quad W = \text{Cov}(\tau, \tau)^{-1} \]

- Calculate a solution from null space that represents the high frequency components (here: characteristic peaks):

\[ w_N = \sum_{b=R+1}^{B} (v_b^T \cdot w_H) v_b \]

- Add the solution from null space to the solution from range:

\[ w = w_R + w_N \]
Materials and Methods

Prior Truncated Singular Value Decomposition (PTSVD)

• We model the prior spectrum:

\[ w_H(h) = \sum_{p=1}^{P} h_p e_p \]

• Cost function

\[ C(h) = \left\| w_L(h) \wedge 0 \right\|_2^2 + \lambda \left\| \nabla \cdot w_L(h) \right\|_2^2 \]

Non-negativity
Smoothness

• Iteration schema:

TSVD solution \( w_R \)

PTSVD solution \( w \)

High frequency spectrum \( w_H \)

Low frequency spectrum \( w_L \)

convergence

Evaluate \( C(h) \)
Materials and Methods

Simulation / Measurement Study

• Simulation conditions:
  – 150 kV tungsten target spectrum simulated according to Tucker et al.
  – Spectrum estimation from 28 aluminum (Al) attenuators with lengths ranging from 0.5 mm to 132.5 mm
  – Poisson noise is added to the Al transmission data for varying numbers of incident photons $N_0$
  – Noiseless simulations of polyoxymethylene (POM) with continuous attenuation length for validation

• Measurement conditions:
  – Experimental setup consisting of a 150 kV transmission x-ray tube and a flat detector
  – 28 measurements of Al and POM attenuators with attenuation lengths ranging from 0.5 mm to 132.5 mm
  – Material for spectrum estimation: Al
  – Material for spectrum validation: POM
Results

Noiseless Simulated Data

\[
\frac{\Delta d}{d} = \frac{|d' - d|}{d}
\]
Results

Noisy Simulated Data

\[ N_0 = 1 \times 10^{12} \]
Results

Noisy Simulated Data

\[ N_0 = 1 \times 10^{10} \]
Results

Noisy Simulated Data

$N_0 = 1 \times 10^8$
Results

Noisy Simulated Data

\[ N_0 = 1 \times 10^6 \]
Results

Measured Data

\( N_0 \approx 1 \times 10^{10} \)
• PTSVD overcomes the limitations of TSVD by incorporating prior information about the statistical nature of the transmission data and about the high frequency components of the spectrum.
• PTSVD is less prone to noise compared to TSVD.
• Simulations show that for accurate transmission data PTSVD leads to smaller length errors compared to EM.
• Effects that limit the accuracy of transmission measurements: quantum noise, electronic noise, scattered radiation, image lag, quantization errors, dynamic range, ...
Thank You!

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This presentation will soon be available at www.dkfz.de/ct.

Job opportunities through DKFZ’s international PhD or Postdoctoral Fellowship programs (www.dkfz.de), or directly through Marc Kachelrieß (marc.kachelriess@dkfz.de).

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