Beam Hardening and Scatter Removal with Empirical Cupping Correction for Primary Modulation (ECCP)

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Aim

- To reduce scatter using the technique of primary modulation
- To correct for beam hardening artifacts in case of spatially strongly varying x-ray spectra
The detected spectrum is a function of the line of integration $L$:

$$q(L) = -\ln \int dE \ w(L, E) \ e^{-\int dL \ \mu(r, E)}$$
Primary Modulation Scatter Estimation (PMSE)

Basic Idea

Key hypothesis: “Low-frequency components dominate the scatter distribution even if high-frequency components are present in the incident x-ray intensity distribution.”

The measurement with a modulator can expressed in Fourier space with:

\[ P'(\omega) = \frac{1 + \alpha}{2} P(\omega) + \frac{1 - \alpha}{2} P(\omega - \pi) + S(\omega), \]

where \( P \) and \( S \) denote the Fourier transforms of primary and scatter, respectively, and \( \omega \in [-\pi, \pi] \times [-\pi, \pi] \) is the 2D coordinate of \((\omega_x, \omega_y)\) in the Fourier domain. Parameter \( \alpha \in (0, 1) \) is the transmission factor of the modulator blocker,

Scatter \( S \) can be estimated by

\[ S_{est}(\omega) = P'(\omega)H(\omega) - \frac{1 + \alpha}{1 - \alpha} P'(\omega - \pi)H(\omega). \]

with \( H(\omega) \) being a low-pass filter.

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Modulator

Photograph of the copper modulator

Projection image of the modulator

$M(u, v)$
**Primary Modulator Introduces Beam Hardening**

- The primary modulator introduces high frequency variations of the incident x-ray spectrum.
- These variations show up as ring artifacts in the reconstructed images\(^1,2,3\).

ECCP Calibration Procedure

• Beam hardening can be corrected as

\[ p(\alpha, u, v) = \sum_{ij} c_{ij} M^i(u, v) q^j(\alpha, u, v) \]

• Let us define basis volumes as

\[ f_{ij}(r) = R^{-1} M^i(u, v) q^j(\alpha, u, v) \]

such that

\[ f(r) = \sum_{ij} c_{ij} f_{ij}(r) \]

• To determine \( c_{ij} \) minimize

\[ \int d^3r \, w(r) \left( f(r) - t(r) \right)^2 \]

which is the weighted distance between the volume \( f(r) \) and a template volume \( t(r) \), that is a binary version of the uncorrected calibration phantom.
Central slices through nine different basis volumes

\[ f_{ij} = R^{-1} M^i q^j \]
Materials

- Acquisition with a tabletop system
  - Water Phantom (calibration)
  - Catphan Phantom (slightly larger)
  - Thorax Phantom (significantly larger)
- Measurement with and without primary modulation (0.21 mm thick copper checkerboard pattern)
- Applying ECCP and PMSE
- Compare to a slit scan measurement
Results
Correction of the Catphan Phantom

Measurement without Modulator  Measurement with Modulator  ECCP–corrected

C = 0 HU, W = 500 HU
Results
Combined correction with PMSE and ECCP

Measurement without Modulator
PMSE+ECCP–corrected
Slitscan without modulator

C = 0 HU, W = 500 HU
Results

Combined correction with PMSE and ECCP
Results
Correction of the Thorax Phantom

Measurement without Modulator  Measurement with Modulator  ECCP–corrected

C = 0 HU, W = 1000 HU
Conclusions on ECCP

• ECCP is dedicated to rapidly varying spectra, e.g. as caused by
  – primary modulators
  – the heel effect
  – wedge filters
  – scratches in the filtration
  – varying sensitivity of the detector pixels
  – …..

• ECCP is an efficient and simple way to correct for first order beam hardening.

• ECCP can be combined with PMSE to nearly completely eliminate beam hardening and scatter artifacts.
Thank You!

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